

Arithmetic with Statistics

March 1997

1. The ratio of the number of games won to the number of games lost by a certain team is 275%. To the nearest whole percent, what percent of its games did the team lose?
2. 160 expressed as the sum of two primes a and b . Find a and b so that $b - a$ is as small as possible.
3. A communications channel can transmit 150 chunks per second. If each block consists of 512 chunks, how much time will it take to send 60 blocks of data? Give exact answer.

Arithmetic with Statistics

March 1999

1. Meg and Peg both play on the basketball team. Meg made 38 out of 50 shots during the first half of the season, and Peg made 30 of 40 attempts. During the second half of the season, Meg made 42 of 70 attempts and Peg made 14 out of 24. Who had the better average during the entire season and by how much more? Give answer as a fraction in simplest form or as a decimal rounded to the nearest 1000th.
2. A list consisting of seven positive, but not necessarily distinct, integers has an arithmetic mean of 8 and a median of 11. What was the largest value that the range can assume for this list? Where range is a difference between the largest and smallest number.
3. Among 100 applicants for a technical position, 10 had never taken a course in chemistry or physics. Seventy-five had taken at least one chemistry course and 83 had taken at least one physics course. How many had taken both a chemistry and a physics course?

Arithmetic with Statistics

March 2000

1. Find the product of $413_5 \times 32_5$ in base five.
2. From the following data, find A such that the mean of set B equals the median of set B .
Set $B = \{1, 1, 3, A, 1, 0, -4\}$
3. Find the sum of the mean, median and mode of the following data: $\sqrt{2}, \sqrt[3]{2}, \frac{p}{3}, \sqrt[4]{3}, \sqrt[4]{4}, \sqrt[5]{5}$
Round answer to the nearest 100th.

Arithmetic with Statistics(Calculators)

March 2001

1. Determine the product of the mean, median and mode for 2, 5, 4, 7, 3, 5, 9

2. The variance for of a list of samples is defined as $V = \frac{\sum_{i=1}^n (x_i)^2 - \frac{\left(\sum_{i=1}^n x_i\right)^2}{n}}{n}$. Find the variance for the list: 1, 5, 7, 8, 4

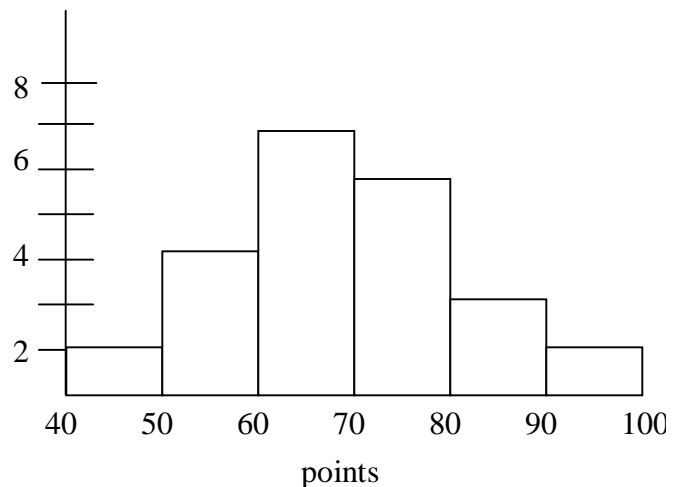
3. How many 3-digit numbers in base 5 are 3-digit numbers in base 8?

Arithmetic with Statistics(Calculators)

March 2003

1. On a certain day Ruth has 200 hits in 600 bats for a batting average of $.3333\dots$. Williams on the other hand has 199 hits in 600 at bats. On the next day, Ruth goes 8 hits for 8 at bats, while Williams goes 11 for 13. At the end of the day, what is the difference between the batting averages of the two players? Round your answer to the nearest hundred thousandth.

2. The histogram shows the frequency distribution for the number of points scored in a game by the WHS basketball team over the course of a season. Assuming the points scored in each game is the mean of the range it falls in, what is the standard deviation to the nearest hundred?



3. A pile of melons weighed 100 pounds and was 99 percent water by weight. On a hot day the pile dried out until it was only 98 percent water. How much did the pile weigh at this point?

Arithmetic with Statistics(Calculators)

March 2004

1. Find A in the whole number $215,7A2$ such that the whole number is divisible by 36.
2. Find the number N such that the data 5, 2, 3, 3, 4, N , 4, 4, 3, would have a mean which would be one more than the median of the data.
3. From the list below Jill added a positive whole number so that the median and the mean of the 15 numbers is the same. Find the variance to the nearest 1000th of the 15 numbers.
32, 56, 44, 49, 39, 33, 58, 63, 55, 42, 38, 53, 43, 60

Arithmetic with Statistics(Calculators)

March 2005

1. You have a mean of 100 after your first ten Latin quizzes. For how many quizzes in a row beyond these ten can you get a zero for each and still maintain a mean of at least 60?
2. Alice and Bert take a self-paced summer economics course lasting 2 months. They make a complicated bet: whoever has the higher test mean in each of the months wins an ice cream cone for the month, and whoever has the higher test mean for the whole course wins 3 more, for a total of 5 ice cream cones. If the results are as shown in the table, how many ice cream cones does Alice win?

Person	Alice	Bert	Alice	Bert
Month	July	July	August	August
Scores	78, 85, 80	83,81,82,74	80,62,75,72,76	66,73,70,80

3. M is the data set 50,81,75,63,65,48,72, X , where X is an integer. The mean of M is an integer, and is greater than or equal to the smallest data value of M and less than or equal to the largest data value of M , neither of which is X . List all possible values of X that would cause set M to have a unique mode.

Arithmetic with Statistics(Calculators)

March 2006

1. If the following fractions were arranged in descending order, which would be the first and which would be the last? Label your answer.

$$\frac{23}{31}, \frac{35}{49}, \frac{47}{63}, \frac{51}{69}, \frac{63}{89}$$

2. The mean of a set of 5 numbers is 12. 4 numbers of the set are 9, 12, 14, and 11. Find the mean of the mode and the median of the set.
3. In a dart game the following numbers of points were hit the given number of times: 8 pts - 12 times, 12 pts – 15 times, 20 pts – 16 times, and 15 pts – 9 times. If the mean is rounded to the nearest whole number, find the sum of the mean, median, and mode of the number of points.

Arithmetic with Statistics(Calculators)

March 2007

1. The Spiranic High School's girl's basketball team finished 6 games with 37 points, 5 games with 42 points, 4 games with 53 points, and 3 games with 49 points. If the mean is rounded to the nearest whole number, what is the sum of the mean, median and mode of these game scores?
2. Find the set of six integers such that the mean is 19, the mode is 21, the median is 20, the range is 10 and the maximum is 25. List the numbers in increasing order.
3. A set of n numbers has a mean, or average, of n . A proper subset of this set has m elements ($m < n$) whose mean is m . What is the mean of the remaining $n - m$ numbers? Be sure your answer is in simplest form.

Conics

March 1989

1. Find the equations of the asymptotes of the hyperbola whose equation is $y^2 - 4x^2 - 24x - 52 = 0$.
2. Find the equation for the locus of all points P interior to the region bounded by $x^2 + y^2 = 1$ such that the sum of the squares of the distances from P to the endpoints of the diameter is 3. State your answer in simplest form.
3. Find the coordinates (x, y) for the points of intersection of the line $L: 3x - 2y = 24$ and the parabola whose directrix has the equation $y = -6$ and whose focus F is $(2, 0)$.

Conics

March 1990

1. Find the distance between the foci of the conic whose equation is $49x^2 + 9y^2 = 441$. State your answer in simplest radical form.
2. Find the equation of the hyperbola whose vertices are $P(3, -3)$ and $Q(3, 7)$ if the point $(9, 15)$ lies on the hyperbola.
3. Find the equation of the locus of the centers of all circles with a diameter of 5 units that are externally tangent to the curve whose equation is $4x^2 + 4y^2 - 16x + 24y + 43 = 0$. State your answer in $(x - a)^2 + (y - b)^2 = k$ form.

Conics

March 1992

1. Find the radius of the circle $x^2 + y^2 - 4x - 6y = -9$.
2. Find the equations of the asymptotes of $4x^2 - 9y^2 + 8x + 90y - 257 = 0$. Leave answer in $Ax + By + C = 0$.
3. The foci of the conic are $(7, 3)$ and $(7, -13)$. Find the equation of the conic if its eccentricity is $\frac{8}{17}$.

Conics

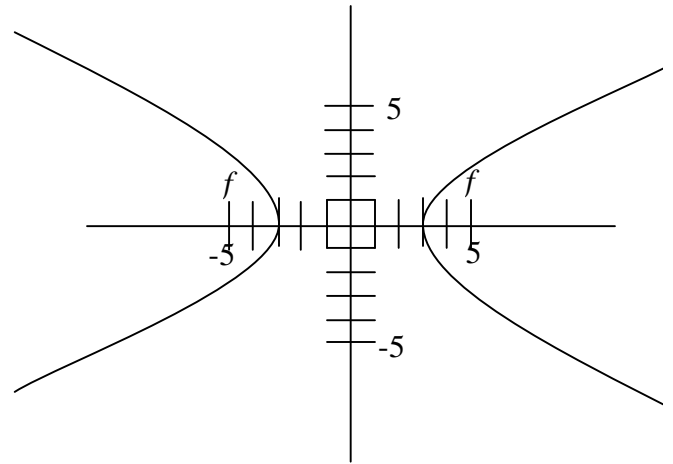
March 1993

1. Find the distance between the foci of the conic section whose equation is $4x^2 - 9y^2 + 16x + 54y - 101 = 0$.
2. An ellipse is inscribed in a rectangle. The perimeter of the rectangle is 20 and the coordinates of one side of the rectangle are $(-4, 3)$ and $(-4, -1)$. What are the possible equations of the ellipse?
3. Write the equation of the line containing the longest chord of the conic section $x^2 + y^2 - 8x + 6y - 15 = 0$, if one point of intersection of the chord and the circle is $(-2, -1)$. Express your answer in $y = mx + b$ form.

Conics

March 1994

1. The standard form for the equation of all conics is $Ax^2 + By^2 + Cx + Dy + E = 0$. Find the sum of $A + B + C + D + E$ for the conic section below, where A, B, C, D, E are relatively prime integers.



2. Find the equation of the parabola whose vertex is $(-2, 3)$ and whose latus rectum has endpoints of $\left(-\frac{1}{2}, 0\right)$ and $\left(-\frac{1}{2}, 6\right)$.

3. The points of intersection of the equations $y = x^2, y = -x^2, x = 2y^2$ and $x = -2y^2$ are used to form a circle with its center. Find the equation of the circle. Express R^2 to the nearest 1000th.

Conics

March 1996

1. Find the vertices of the hyperbola $4x^2 - y^2 - 24x - 12y - 16 = 0$.
2. Find the equation of the parabola with axis of symmetry parallel to the y-axis, whose vertex is (3, -1) and contains the point (9, -4). Express answer in either of the normally accepted forms.
3. An ellipse which is tangent to both the x and y-axis has an eccentricity of $\frac{4}{5}$. Find its equation in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, if one of its foci is the center of the $x^2 + y^2 - 2x + 6y - 39 = 0$.

Conics

March 1997

1. Find the distance between the vertices of the conic section whose equation is $16y^2 - 64y - 9x^2 - 54x = 161$.
2. The ellipse $4x^2 + 9y^2 = 36$ and the circle $x^2 + (y - 1)^2 = 9$ intersect at three points. Find the coordinates of the intersection with the smallest y-coordinates.
3. The latus rectum of the parabola P is the horizontal axis of the circle $x^2 + y^2 - 6x + 2y - 6 = 0$. Find all possible equation for P .

Conics

March 1998

1. Find the foci of the conic section $4x^2 - 16y^2 = -256$.
2. Find the focus of the new parabola when the parabola $x^2 - 8x - 8y = 0$ is rotated on its vertex 90° to open left.
3. One of the endpoints of the diameter of a circle is $(15, 18)$. The circle is tangent to the line $3x + 4y = 17$ at the other endpoint. Determine the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$.

Conics

March 1999

1. Find the focus of the parabola $x^2 - 6x - 4y + 1 = 0$.
2. Find the equation of the ellipse in $Ax^2 + By^2 + Cx + Dy + E = 0$ form, if its major axis is 10 units long and its foci are $(8, 2)$ and $(0, 2)$.
3. Find the locus of a point that moves so that its distance from the point $(4, 0)$ is always twice its distance from the line whose equation is $x = 1$.

Conics

March 2000

1. Find the center of the circle whose equation is $x^2 + y^2 - y - 4x - 4\frac{3}{4} = 0$.
2. An ellipse with center (4, 6) is tangent to the x-axis. Find its equation if its eccentricity is $\frac{\sqrt{3}}{2}$, and its major axis is parallel to the x-axis.
3. The equation of the asymptotes of a hyperbola are $x - 2y = 11$ and $x + 2y = -1$. Find its equation if one of the endpoints of the conjugate axis is (5, -5).

Conics

March 2001

1. Find the foci of the ellipse $16x^2 + 4y^2 = 64$.
2. Find the equation of the hyperbola whose vertices are (5, 5) and (-11, 5) and whose foci are (-20,5) and (14,5).
3. There are two tangent segments from the point (10, 7) to the circle $x^2 + y^2 + 6x - 14y + 33 = 0$. Find the points of tangency.

Conics

March 2002

1. Find the vertices of the ellipse whose equation is $x^2 + 4y^2 - 16x - 24y + 84 = 0$.
2. The vertex and focus of a conic centered at the origin are $(0, 4)$ and $(0, 2\sqrt{13})$ respectively. Find the equation of the conic.
3. The circle with center $(12, 5)$ is tangent to the line $7x - 24y = 89$. Find the equation of the circle.

Conics

March 2003

1. Find the distance between the foci of the ellipse $\frac{x^2}{225} + \frac{y^2}{625} = 1$.
2. Given the hyperbola centered at the origin with a conjugate axis of length 10 and a y-intercept of 8, find its eccentricity. Express in simplest form.
3. The two parabolas $y^2 - 2x - 6y + 1 = 0$ and $y^2 + 2x - 6y - 7 = 0$ when graphed enclose a region which appears to be elliptical in shape. Use the axes of the elliptical shape and their point of intersection as the center to form an equation of an ellipse. Express answer in $Ax^2 + By^2 + Cx + Dy + E = 0$ form, where $A, B, C, D,$ and E are integers and their GCF = 1.

Conics

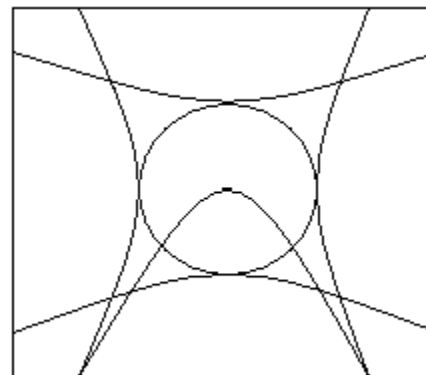
March 2004

1. Find the center of the circle $x^2 + y^2 - 4x - 6y + 7 = 0$.
2. Find the equation of the parabola in the form $x^2 + Ax + By + c = 0$ or $y^2 + Ax + By + C = 0$, if its directrix is $y = 3$ and focus is $(-3, -7)$.
3. Find the equation of the circle whose center is the center of the hyperbola $7y^2 - 9x^2 - 28y - 54x = 116$, and two of whose points are the foci of the hyperbola.

Conics(No Calculators)

March 2005

1. Find the value of k so that the equation $x^2 + y^2 - 6x - 8y + k = 0$ is the equation of a circle of radius 8.
2. The drawing, entitled “MAML Happy Face”, is shown on the grid $|x| \leq 3$ and $|y| \leq 3$. It consists of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ and its 90 degree rotation about the origin, a circle tangent to both the hyperbolas at 2 points, and a parabola with vertex at the center of the circle that intersects the hyperbola at the bottom of the graph, as shown. Find the equation of the parabola.



3. Ellipse 1 has a semi-major axis of 2 that is parallel to the y-axis and a semi-minor axis of 1 that is parallel to the x-axis. Ellipse 1 is in the 1st quadrant and is tangent to both axes. Ellipse 2 has the same center and eccentricity as ellipse 1 and is internally tangent to it. The major axis of ellipse 2 is perpendicular to the major axis of ellipse 1. If the equation of ellipse 2 is written in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, find $h+k+a+b$ as a number in simplest form.

Conics(No Calculators)

March 2006

1. Find the endpoints of the major axis of the ellipse $x^2 + 4y^2 = 16$.
2. The center of an ellipse is $(-2, 5)$. The ellipse is tangent to both axes. Find the equation of the ellipse in the form of $Ax^2 + By^2 + Cx + Dy + E = 0$.
3. Find the equation of the locus of points which are the same distance from the point $(9, 5)$ as they are from the line $x = -1$.

Conics(No Calculators)

March 2007

1. Find the center and radius of the circle $x^2 - 4x + y^2 + 10y = 7$.
2. Tangent segments are drawn from the point $(25, 0)$ to circles centered at the origin. The lengths of the tangent segments and radii are natural numbers. If the circle equations are written in the form of $x^2 + y^2 = r^2$, find the sum of all r that produces circles of these specifications.
3. Find the equation, in $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ form, for the ellipse described as the set of all points p whose distances from the points $(7, 2)$ and $(3, 2)$ add up to 12.

Trigonometric Equations and Identities

March 1989

1. Find the $\cos 75^\circ$. State your answer in simplest radical form.

2. Find K where K is a rational number in lowest terms if

$$4 \cos^2 A + \sin^2 A = 1 + K \sin^2 A.$$

3. Solve for A if $0 \leq A < 2\pi$: $\tan A + \sec A + 1 = 0$.

Trigonometric Equations and Identities

March 1990

1. Reduce $\frac{\tan X - \cot X}{\tan X + \cot X}$ to an expression whose trigonometry part involves only $\sin X$.

2. Solve: $\arcsin X + \arccos(1 - X) = 0$.

3. Find all values of X , $0 \leq X < 2\pi$ so that $\sin X - \sqrt{3} \cos X = 1$.

Trigonometric Equations and Identities

March 1992

1. If a ladder has to be placed such that the angle it makes with the ground is at most 60° , how high can a 30 foot ladder reach?

2. Solve: $\sin^2 x + 4 \csc 3x^2 + 2 = 1$

3. Solve: $\sin 2x = \sqrt{2} \cos x$ for $0 \leq x \leq 2\pi$

Trigonometric Equations and Identities

March 1993

1. Find the exact value of the following. Express answer in simplest form with a rational denominator.

$$\frac{\tan 210^\circ - \cos 300^\circ}{\csc 120^\circ - \cot 150^\circ}$$

2. In obtuse triangle WHY , $HY = 42$ and $YW = 21\sqrt{6}$. If $m\angle W = 45^\circ$, find all possible measure(s) of the exact value of $m\angle H$.

3. Solve for x , where $0^\circ \leq x < 360^\circ$. Express answer(s) to the nearest minute.

$$\tan x - \tan x = 4 \tan x - 3$$

Trigonometric Equations and Identities

March 1994

1. Find the radian measure in terms of π , for which $\sqrt{3} \csc x + 2 = 0$ and $0 \leq x \leq 2\pi$.

2. Find all value(s) of x such that: $0 \leq x \leq 180$ and $2 \sin x \cos x + \cos 2x = 1$.

3. Determine all values of x , where $0 \leq x < 360$ for which $\sin 2x \leq \tan x$.

Trigonometric Equations and Identities

March 1995

1. Find all solutions of the equation $\csc^2 x - 2 = 0$, where $0 \leq x < 2\pi$. Express answer(s) in terms of π .

2. Evaluate the expression $\cos \left(\arctan 1 + \arccos x \right)$. Express your answer as a single fraction in terms of x .

3. Rewrite the following expression in simplest form so that it is not in fractional form and involves only sine and cosine. $\frac{\sin^3 y}{1 - \cos y} + \frac{\cos^3 y}{1 + \sin y}$

Trigonometric Equations and Identities

March 1996

1. Find all values of x such that $0^\circ \leq x < 360^\circ$ for which $\sin 70^\circ = \cos x$.
2. Express $\frac{\tan x}{1 + \sec x} + \frac{1 + \sec x}{\tan x}$ in terms involving only \csc . Express in simplest possible form.
3. Find all solutions for x which $0^\circ \leq x < 360^\circ$ and $2 \csc^2 x - 3 \cot x - 7 = 0$. Find angles to the nearest minute.

Trigonometric Equations and Identities

March 1997

1. Rewrite $\frac{\csc x + \sec x}{1 + \cot x}$ as an expression with a single trig function in simplest form, without a denominator.
2. Find the sum of all solutions to the equation $2 \tan x - 4 \sin x \cos x = 0$ for $0^\circ \leq x < 360^\circ$.
3. Find all values of x such that $x^3 + 27i = 0$.

Trigonometric Equations and Identities

March 1998

1. Find all θ for which $\sin 2q - \cos q = 0$, if $0^\circ \leq q < 360^\circ$.
2. Change $\frac{\sin q}{\csc q - 1} + \frac{\sin q}{\csc q + 1}$ into an expression with only $\tan \theta$. Express in simplest form.
3. Find all x , where $0^\circ \leq x < 360^\circ$ for which $2 \cos x - 2 \sin x + 4 \sin x \cos x = 1$.

Trigonometric Equations and Identities

March 1999

1. Express $\frac{1}{\sec x} (\tan x + \cot x)$ as a single trig function without a fraction in simplest form.
2. If $\sin a = \frac{3\sqrt{10}}{10}$ where $0 < a < \pi/2$, find $\sin 2a$. Express answer in simplest form.
3. Solve: $\cos^4 2q - \sin^4 2q = 1$.

Trigonometric Equations and Identities

March 2001

1. If $0^\circ \leq q < 360^\circ$, for how many values of θ is the equation $5\sin^2 q + \sin q - 4 = 0$ is true?

2. Express $\frac{\sec x + \tan x}{\sin x} \csc x$ as a single trigonometric function without a denominator.

3. If $\sin 4x = A \sin x \cos x + B \sin^3 x \cos x$ for all values of x , find $A + B$.

Trigonometric Equations and Identities

March 2002

1. Express $\sin^2 x + \cos^2 x + \tan^2 x + \sec^2 x$ as a single trig function in simplest form.

2. Find all values θ for which $\cos^2 q - \sin q \cos q = 0$, if $0^\circ \leq q < 360^\circ$.

3. Find all value(s) of x such that $\sec^2 x + 3 \csc^2 x = 8$, where $0 \leq x < 2\pi$.

Trigonometric Equations and Identities

March 2003

1. Solve for x , where $0^\circ \leq x \leq 180^\circ$, and $\sin x \cos x \tan x \sec x \csc x = 1$.
2. Evaluate: $\cos \frac{5p}{12} \cos \frac{p}{12} + \sin \frac{5p}{12} \sin \frac{p}{12}$. Express your answer as a fraction in simplest form.
3. Find all values of x , where $0^\circ \leq x \leq 360^\circ$, if $\sin x + \cos x = \frac{\sqrt{2}}{2}$

Trigonometric Equations and Identities

March 2004

1. Find all x , where $-360^\circ \leq x \leq 360^\circ$, such that $\cos x = \frac{1}{2}$.
2. If $\cos \frac{p}{8} = n$ and $\sin \frac{p}{8} = m$, find the product of $m \cdot n$ in simplest radical form.
3. Find all value(s) of x either in degrees or radians where $0^\circ \leq x \leq 180^\circ$ or $0 \leq x \leq p$, if

$$2 \cos 2x \cos x - 8 \sin x \cos \frac{1}{2} x \cos x \sin \frac{1}{2} x + 1 = 0$$

Trigonometric Equations and Identities (No Calculators)

March 2005

1. Find q if $0^\circ \leq q \leq 360^\circ$ and $\sec q = \csc q$.
2. How many solutions to the equation $2 \cos^2 f + 3 \cos f + 1 = 0$ exist in the interval $-360^\circ \leq f \leq 360^\circ$?

3. Express $\frac{\cos 2q (\tan^2 q + 1) \sin 2q}{\cos q} + 2 \tan q$ in terms of only $\sin q$ in simplest form.

Trigonometric Equations and Identities (No Calculators)

March 2006

1. Find all values of q , where $-180^\circ \leq q \leq 180^\circ$ and $\sqrt{3} \sec q - 2 = 0$.
2. Express the following only in terms of $\sin f$ in simplest form: $\frac{\sec f \sin f}{\tan f + \cot f}$
3. Find all values of q for which $-180^\circ \leq q \leq 180^\circ$ and $2 \sin q - 2 \cos q = 1 - 2 \sin 2q$.

Trigonometric Equations and Identities (No Calculators)

March 2007

1. Simplify, reducing the expression to a real number or to an expression involving exactly one of the six trig functions.

$$\frac{1}{\sec x}(\tan x + \cot x)$$

2. Solve for θ : $\cos^4 2q - \sin^2 2q = 1$

3. Find $\tan 105^\circ$. Give an exact answer in simplest form.

Trigonometric Equations and Identities (No Calculators)

March 2008

1. Solve for x , where $0 \leq x \leq 2\pi$ and $3 \sin x = -1.5$.

2. If $\cos x = .8$ and $0^\circ \leq x \leq 90^\circ$, find the value of $\cos 2x + \sin 2x$.

3. Find all value(s) of x such that $\sin \frac{1}{2}x - \cos x = 0$ and $0^\circ \leq x < 360^\circ$

Algebraic Fractions with Factoring

November 1988

1. Perform the indicated operations. Express your answer as a rational expression in lowest terms.

$$\frac{1}{x^2 + x} - \frac{4}{x^2 - 1} + \frac{1}{x^2 - x}$$

2. Solve for x : $\frac{3x+2}{x-1} + \frac{2x}{x+1} = \frac{7x+3}{x^2-1}$

3. Find the smallest positive fraction $\frac{x}{y}$, in lowest terms with x and y as integers so that when 7 is added to both the numerator and the denominator of the fraction, the value of this new fraction is equal to the value of the fraction obtained by subtracting 2 from the numerator and 8 from the denominator of the original fraction.

Algebraic Fractions with Factoring

November 1989

1. Simplify: $\frac{\frac{1}{w^2 - w} + \frac{w+1}{w}}{w-1} - \frac{1}{w}$

2. Find all the value(s) of X , where X is a real number and $5 - \frac{4}{3 - \frac{2}{1 - \frac{1}{X}}} = 1 - \frac{8}{X-3}$

3. Find the value(s) of c if $\frac{a+b}{b+c} = \frac{c+d}{d+a}$

Algebraic Fractions with Factoring

November 1991

1. Simplify: $\frac{x^2 + x - 6}{x^2 - 4x - 5} \div \frac{x + 1}{x - 5} \cdot \frac{x^2 + x}{x^2 - 7x + 10}$

2. Find all the solutions for a such that $\frac{a + 5}{a + 3} + \frac{a + 6}{3 - a} = \frac{a^2 - 6a - 39}{a^2 - 9}$

3. Find all real values of x such that $4 = x + \frac{3 + \frac{1}{x}}{x + 1}$

Algebraic Fractions with Factoring

November 1992

1. Simplify the following to a reduced fraction. Leave answer in factored form.

$$\frac{x + 1}{x^2 - x - 6} - \frac{x + 4}{x^2 - 4x + 3} + \frac{x + 3}{x^2 + x - 2}$$

2. A large water tank can be filled by an intake pipe in 12 hours and be emptied by a drain in 15 hours. If the tank is half full and both pipes are opened, how long will it take to fill the tank?

3. Find A , B , and C , if: $\frac{2x - 3}{x^2 - 4x + 1} = \frac{A}{x - 2} + \frac{B}{x + 2} + \frac{C}{x + 1}$

Algebraic Fractions with Factoring
November 1993

1. Solve for x . Express answer(s) in exact form or to nearest tenth. $x + 5 = \frac{1}{x+1} - 3x + 1$

2. Simplify: $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$

3. Find all value(s) of x such that: $14x - \frac{3}{x^2 - 4} = \frac{6x}{x+2} - \frac{9}{x-2}$

Algebraic Fractions with Factoring
March 1995

1. For what values of A , B , and C does the following equality hold?

$$\frac{2}{x+1} - \frac{Ax^2 + Bx + C}{x^2 - 2x + 3} = \frac{x-1}{x^2 - 2x + 3}$$

2. Luke can do a certain job 5 hours faster than Peter. Luke started the job early one morning. Two hours later Peter came to help. (He needed his beauty sleep.) They finished $4\frac{4}{5}$ hours after Peter came to help. How long would it take Peter to do the job himself?

3. Express in simplest form: $\frac{3x+7}{x+2} - \frac{x^2-7}{x^2+x-2} + \frac{4x-1}{1-x}$

Algebraic Fractions with Factoring
March 1996

1. Express the following in simplest form: $1 + \frac{1}{x - \frac{1}{x}}$

2. Find the solution(s) for: $\frac{1}{x^2 + 5x + 6} + \frac{2}{x + 3} = \frac{5x + 5}{3x + 6}$

3. Determine A , B , and C so that: $\frac{11x^2 - 15x - 72}{x^3 - 9x} = \frac{A}{x + 3} - \frac{B}{x^2 - 3x} + \frac{C}{x}$

Algebraic Fractions with Factoring
March 1997

1. If $(x - 2a)(x + 4) = x^2 - 7x + 3b$, find the value of b .

2. Solve for y : $\frac{2y - 1}{y + 3} + \frac{1}{y^2 + y - 6} = \frac{y - 4}{2 - y}$

3. Sudden Sam's rate on his second trial on the 12 km track was 2 km/min less than the first trial. The total time for both trials was $3\frac{1}{2}$ minutes. Find the time in minutes for the first trial.

Algebraic Fractions with Factoring

March 1998

1. Simplify completely: $\frac{20b^2}{a^2 - 4b^2} - \frac{2a + b}{a - 2b} + \frac{3a - b}{a + 2b}$

2. Factor completely: $x^2a^2 - 4a^2y^2 - 9x^2 + 36y^2$

3. Ben and Alex can complete half of a certain task in 2 hours and 24 minutes. Ben takes 4 hours longer to complete the task alone than does Alex. Find how long it would take Ben to complete the task by himself.

Algebraic Fractions with Factoring

March 1999

1. Simplify: $\frac{x - 5}{2x - 6} - \frac{x - 7}{4x - 12}$

2. The denominator of a fraction is one less than twice the numerator. If seven is added to both the numerator and the denominator, the resulting fraction has a value of seven-tenths. Find the original fraction.

3. Solve: $\frac{r + 2}{2r + 1} = \frac{r}{3} + \frac{3}{4r + 2}$

Algebraic Fractions with Factoring

March 2000

1. Reduce to lowest terms. Leave no ()'s in answer. $\frac{12x^2 - x - 20}{12x^4 - 25x^3 + 12x^2}$

2. Rewrite as a single fraction in simplest form. Leave the denominator in factored form.

$$\frac{1}{x^2 - 3x + 2} + \frac{1}{x^2 - 5x + 6} + \frac{1}{x^3 - x^2 - 4x + 4}$$

3. Find all value(s) of x such that $\frac{\frac{1}{x-3} + \frac{1}{x-2}}{1 - \frac{2x-9}{x^2-5x+6}} = \frac{7}{9}$

Algebraic Fractions with Factoring

March 2001

1. Simplify completely: $\frac{x^2 + 6x + 9}{x^2 + 5x + 6} \cdot \frac{x^2 - 2x - 8}{x^2 - 5x + 4}$ **b-xc**

2. Factor into a product of two binomials: $128xz - 72yz - 48x^2 + 27xy$

3. Solve for x : $\frac{x-3}{x-2} + \frac{x-5}{4-x} = \frac{x^2-2x-8}{x^2-6x+8}$

Algebraic Fractions with Factoring
March 2002

1. Simplify the following and express in lowest terms: $\frac{a-b}{a} + \frac{a+b}{a-b} - \frac{a+b}{b} - \frac{a-b}{a+b}$

2. Find all values of x such that $\frac{1}{3-x} + \frac{5}{x+1} = \frac{8}{x^2-2x-3}$

3. Solve for x , if $\frac{x-3}{x-1} - \frac{x+1}{x+2} = \frac{x-5}{x-2}$

Algebraic Fractions with Factoring
March 2003

1. Express in simplest form: $\frac{x+x^2+x^3+x^4+x^5+x^6+x^7}{x^3+x^4+x^5+x^6+x^7+x^8+x^9}$

2. Find N so that $\frac{N}{x-5} + \frac{3}{x+4} = \frac{10x+13}{x^2-x-20}$

3. Find all values of x that satisfy the equation:

$$\frac{6}{\sqrt{x-8}-9} + \frac{1}{\sqrt{x-8}-4} + \frac{7}{\sqrt{x-8}+4} + \frac{12}{\sqrt{x-8}+9} = 0$$

Algebraic Fractions with Factoring

March 2004

1. Simplify $\frac{x}{x^2 + 5x + 6} + \frac{4}{x^2 + 2x}$

2. Mark can paint a side of a certain house in 3 fewer hours than Todd can paint the same surface. If they work together, it takes them 6 hrs. 40 min. At the same rate, how long does it take Todd to do the job alone?

3. Find the value(s) of x for which $\frac{1}{x - x^3} = \frac{1}{x} + \frac{2}{x - 1}$.

Algebraic Fractions with Factoring

March 2005

1. Simplify $\frac{(x+2)(x+1)(x+3)(x-1)(x^2+4x+4)}{(x-1)(x+1)(x+1)(x-1)(x+1)(x-1)}$

2. Simplify

3. Solve for x . $\frac{x-1}{(x+3)(x-2)(x-3)(x-1)(x-2)(x-1)}$

Algebraic Fractions with Factoring

March 2006

1. Simplify: $\frac{6x-18}{5x-10} \div \frac{2x+4}{3x-6} \cdot \frac{7x+14}{9x-27}$

2. Alex takes 3 hours longer to complete a certain job than Barry does. If they can complete the job by working together in 3.6 hours, how long would it take Alex to complete the job working alone?

3. Find the value(s) of x , such that $\frac{-5x}{1-x} - \frac{2}{x+3} = \frac{8}{x^2+2x-3}$

Algebraic Fractions with Factoring(No Calculators)

March 2007

1. Solve the following equation: $\frac{3}{y-3} = \frac{6}{y^2-9}$

2. Find the values of C and D , if $\frac{C}{x-2} + \frac{D}{x+1} = \frac{6x}{x^2-x-2}$ is an identity.

3. Simplify. Assume no denominator equals zero.

$$\frac{x^3 + 5x^2 + 6x}{4 - y - 4x} \cdot \frac{16x^2 + 8y - y^2 - 16}{x^2(x+2) - x(x+2) - 12(x+2)}$$

Algebraic Fractions with Factoring(No Calculators)

March 2008

1. Solve for x , if $\frac{1}{x} + \frac{3}{x} = x$.

2. Find all value(s) of x such that $\frac{x-20}{x} + \frac{x}{72} = \frac{3}{2}$.

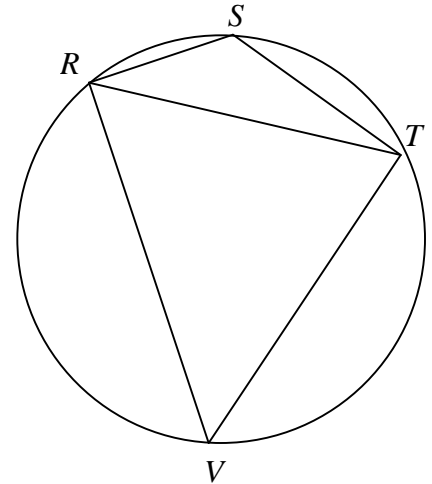
3. Find all real value(s) of x which satisfy the equation:

$$\frac{x}{x^2 - 3x + 2} - \frac{x-1}{x+1} = \frac{x^2 - 5x + 5}{x^2 - 1}$$

Circles and Spheres

March 1989

1. Given: $m\angle SRV = 85^\circ$ $m\text{Arc}RS = 40^\circ$ Find: $m\angle RTV$



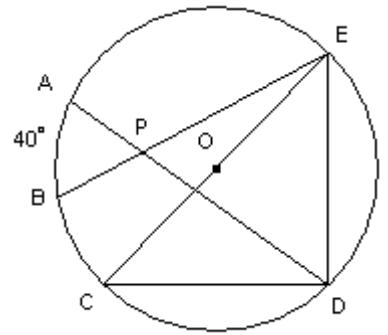
2. A hole 60 cm in diameter is cut in a sheet of plywood, and a sphere 80 cm in diameter is set in this hole. How many centimeters below the surface of the plywood will the sphere extend? Give your answer in simplest radical form.

3. Find the radius, in centimeters, of the inscribed circle of $\triangle ABC$ if $m\overline{AC} = 36\text{cm}$, $m\overline{BC} = 15\text{cm}$, $m\overline{AB} = 39\text{cm}$.

Circles and Spheres

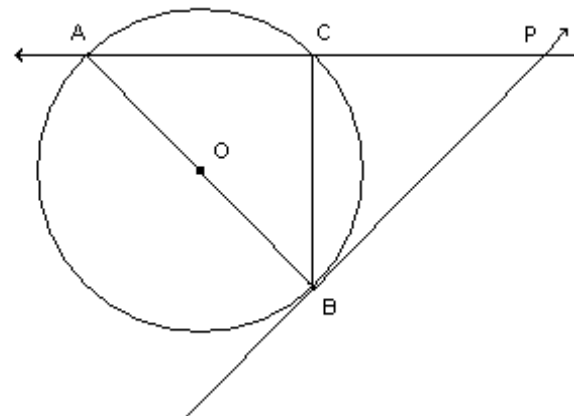
March 1990

- O is the center of the circle.
 $m\text{Arc}AB = 40^\circ$, $m\angle EPD = 70^\circ$. Find $m\angle CED$.



- Two perpendicular chords intersect within a circle. The segments of one chord are six inches and eight inches, and the segments of the other are twelve inches and four inches. Find the area of the circle in square inches. Give your answer in terms of π .

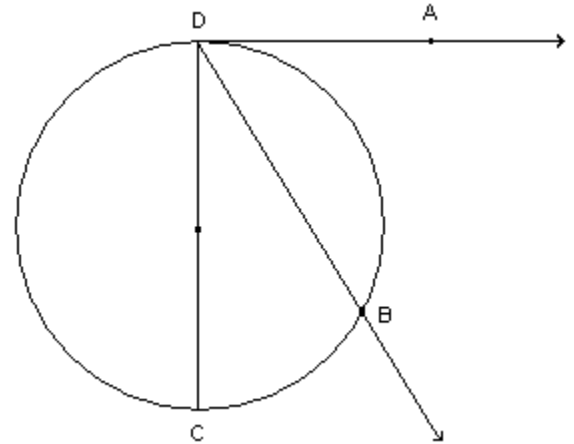
- \overline{PA} is a secant, \overline{PB} is tangent to circle O at B
 $m\overline{PA} = 16$, $m\overline{PB} = 12$
 Find the $m\overline{BC}$. State your answer in simplest radical form.



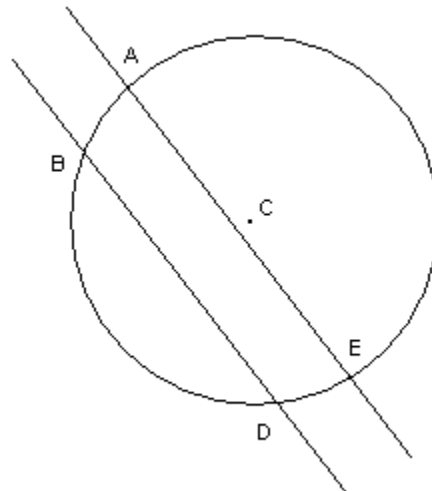
Circles and Spheres

March 1992

1. \overline{AD} is tangent to circle E at D .
 E is the center. If $m\text{Arc}DB = 70^\circ$, find $m\angle EDB$.

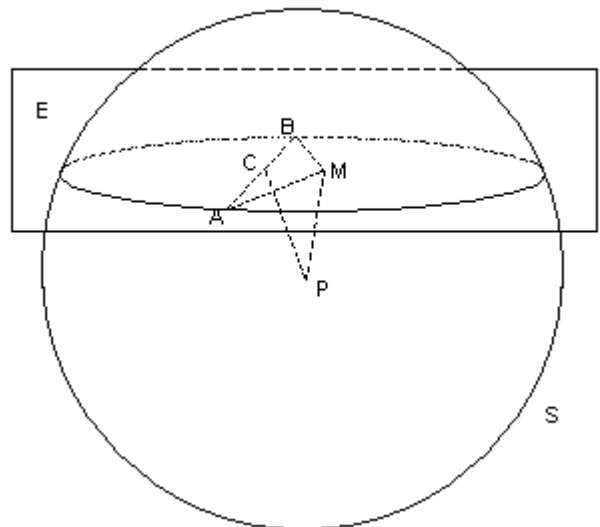


2. Given: The circle centered at C .
 $\overline{AE} \parallel \overline{BD}$, $m\text{Arc}ABE = 40^\circ$
 $m\angle BCE = 90^\circ$. Find $m\angle BCD$.



3. Plane E intersects sphere S . P is the center of S .
 A, B, C, M are in E . A and B are in S .
 $\overline{PM} \perp E$, $\overline{AM} \perp \overline{MB}$, $AC = BC$
 $AM = PM$, $AB = 5$.

Find the length of \overline{PC} .

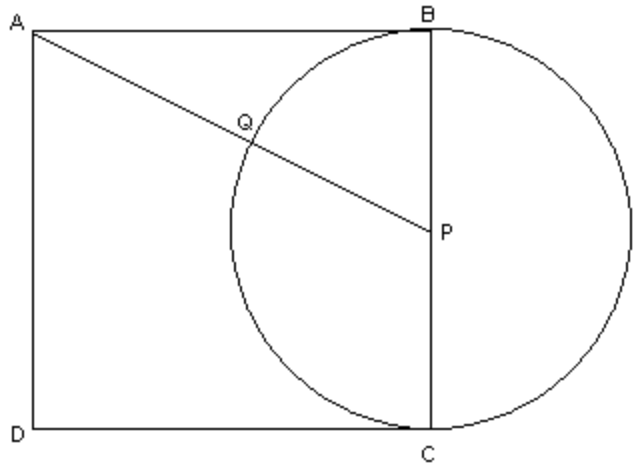


Circles and Spheres

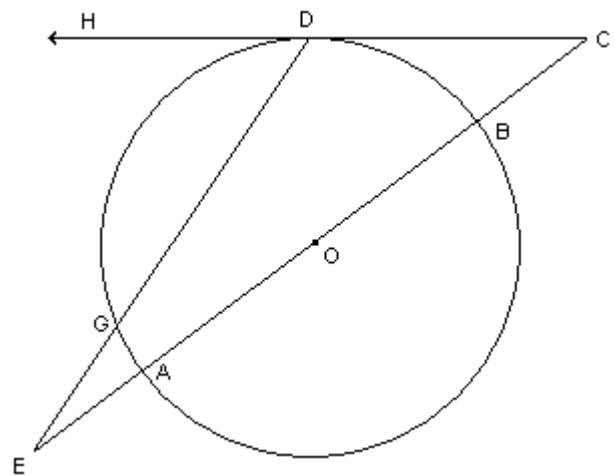
March 1993

- Two concentric circles have diameters 42 cm and 70 cm. How long is a chord of the larger which is tangent to the smaller?

- Side BC of square $ABCD$ is a diameter of circle P . \overline{AP} intersects circle P at Q . If BC is 10, find the length of \overline{AQ}



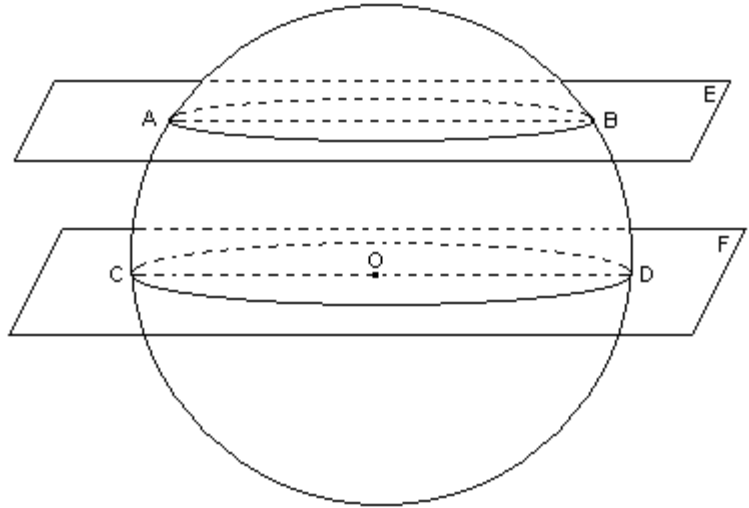
- \overline{CD} is tangent to circle O at D . If $m\angle CDE = 128^\circ$ and $m\angle E = 20^\circ$, Find the measure of arc AG .



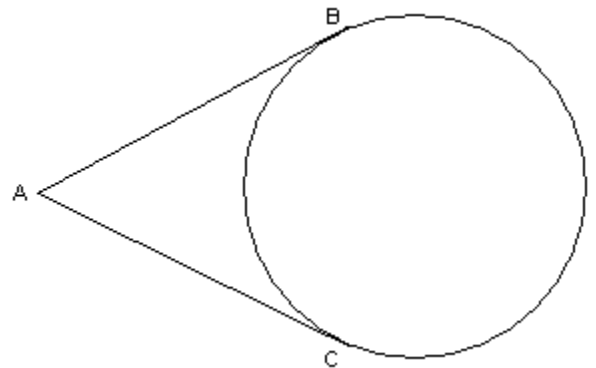
Circles and Spheres

March 1994

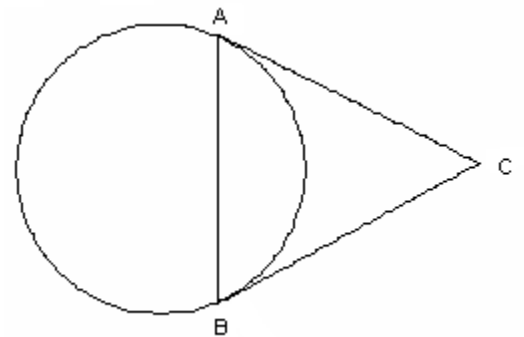
1. Parallel planes E and F intersect sphere O , plane F through the center O . Parallel diameters AB and CD intersect arcs BD and AC on the sphere, each measuring 60 degrees. If the diameter of the sphere is 12 cm long, find the length of AB .



2. The segments AB and AC are tangent to the arc of a circle at points B and C as shown. If the radius of the circle is 8 and arc BC has a measure of 240 degrees, find the perimeter of the figure to the nearest 100th.



3. Triangle ABC is a $17 - 17 - 16$ triangle, where A and B are points of tangency. How far from the circle is point C ? Give answer in exact form as a fraction or as a decimal to the nearest 10th.

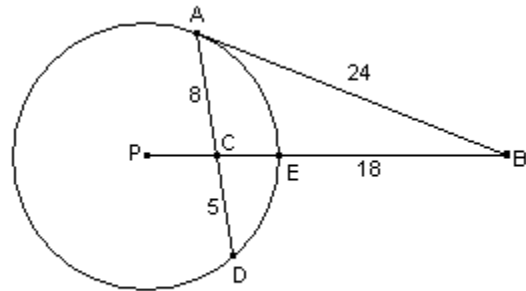


Circles and Spheres

March 1995

1. Two spheres, each having a radius of 13 cm. intersect so that their centers are 24 cm apart. Find the radius of their circle of intersection.

2. BA is tangent to this circle at A . P is the center. $AC = 8$, $CD = 5$, $BE = 18$ and $AB = 24$. Find the length of CE .



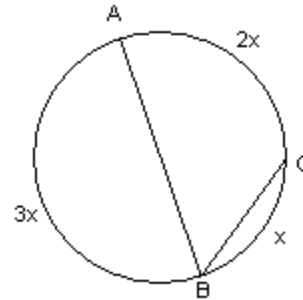
3. The diagonals of a 5 by 12 rectangle are drawn. Circles are inscribed in the two triangles formed by the diagonals and the sides of length 5. How far apart are the centers of the circles?

Circles and Spheres

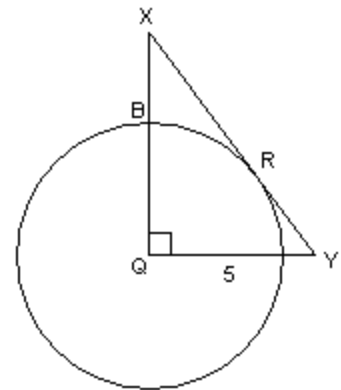
March 1996

1. If the measure of arc $BC = x$,
Measure of arc $AC = 2x$,
Measure of arc $AB = 3x$,

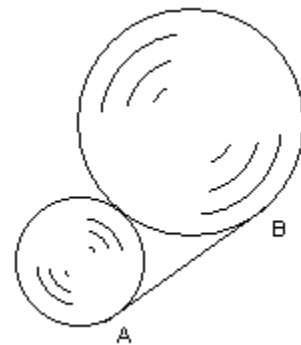
Find $m\angle B$.



2. In circle Q , \overline{XY} is tangent at R .
The radius measures 4 cm.
 $QY = 5$ cm. and $m\angle Q = 90^\circ$.
Find the exact cm length of \overline{BX} .



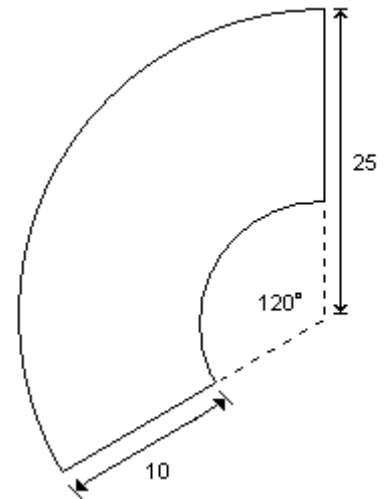
3. The two tangent circles at right
have radii of 15 cm and 12 cm.
Find the length of segment AB
which is tangent to both circles.
(in exact form)



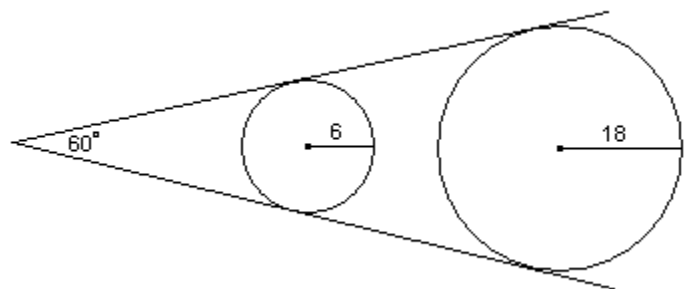
Circles and Spheres

March 1997

1. A secant line and a tangent line intersect at an angle of 51° in the exterior of a circle. If the measures of the arcs intercepted by these lines are in a ratio of 5:2, find the measure of the third arc.
2. Find the area between the 25 in. concentric arcs at right. Give exact answer.



3. The common external tangents to two circular pulleys of radii 6 inches and 18 inches respectively, intersect at an angle of 60° . What is the exact length of the belt required to fit and operate the pulleys?

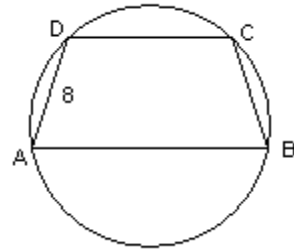


Circles and Spheres

March 1998

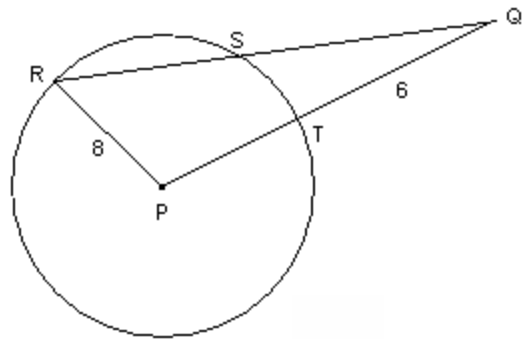
1. $m\text{Arc}AD = m\text{Arc}DC = m\text{Arc}BC$
 $m\text{Arc}AB = 3 \cdot m\text{Arc}AD, \overline{AD} = 8.$

Find the perimeter of quadrilateral $ABCD$.

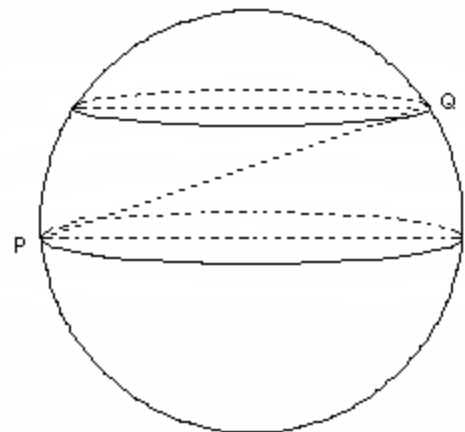


2. Given: $PR = 8, TQ = 6,$ and $QS = 7.$
 P is the center of the circle.

Find the perimeter of $\triangle PQR$ in exact form.



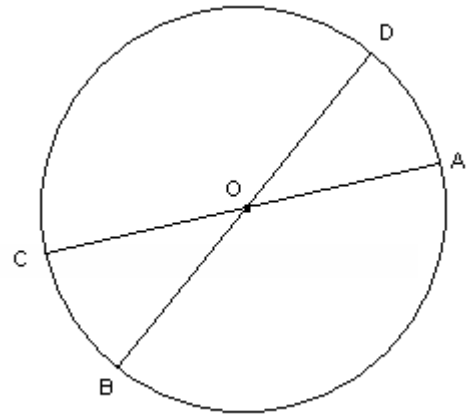
3. In the sphere, the circles have radii of 9 and 15 and are 4 units apart. Determine the length of PQ which passes through the line of centers. Express answer in simplest radical form.



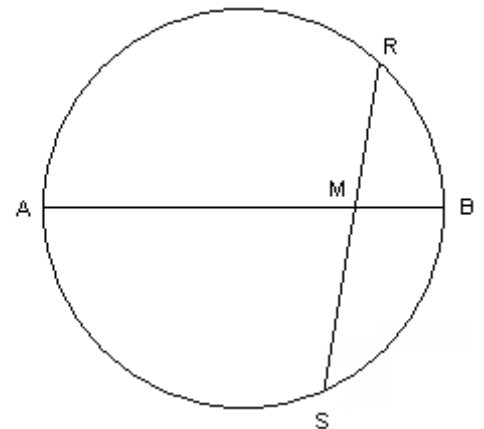
Circles and Spheres

March 1999

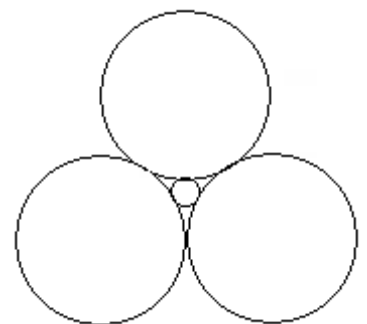
1. Find the length of arc AB given that the radius $OA = 3$ inches and $m\angle DOA = 80^\circ$. Give exact answer or round to nearest hundredth.



2. The diameter AB of circle O intersects chord RS of length 16 at its midpoint M . Find BM if $AB = 20$, and $BM < AM$.



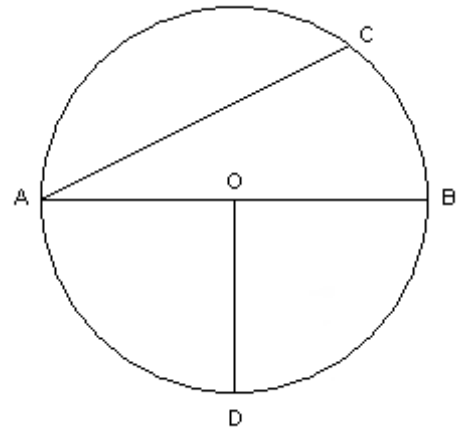
3. In the diagram, three circles of radius 1 cm are each tangent to each other, and a small circle of radius r is tangent to each of the large circles. Determine the exact value of r .



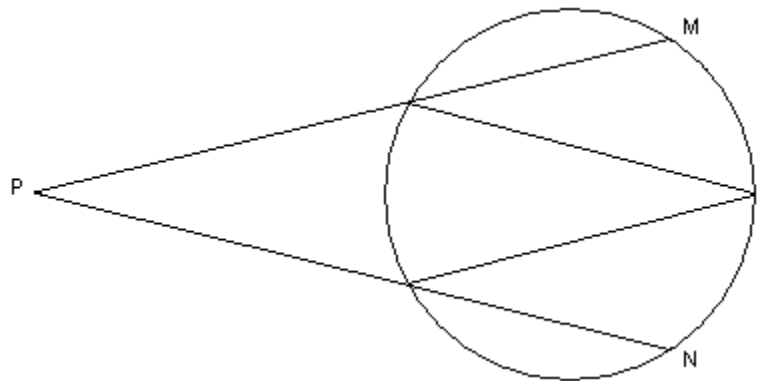
Circles and Spheres

March 2000

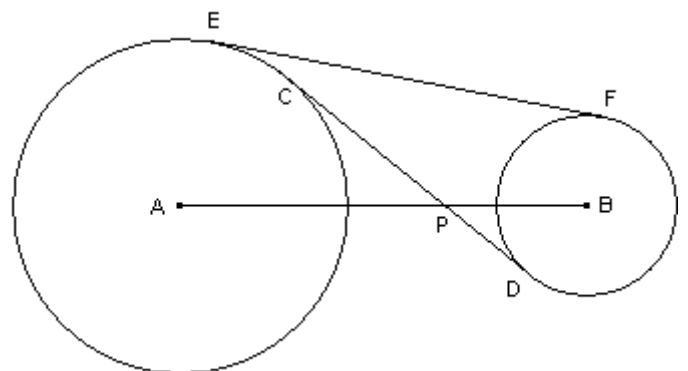
1. $m\angle BAC = 15^\circ$. Diameter AB with center O is 6 cm long. $\overline{DO} \perp \overline{AB}$. Find the length of arc CAD .



2. If $m\text{Arc}MQ = 44$, $m\text{Arc}QN = 36$, find the sum of the measures of $\angle P$ and $\angle Q$.

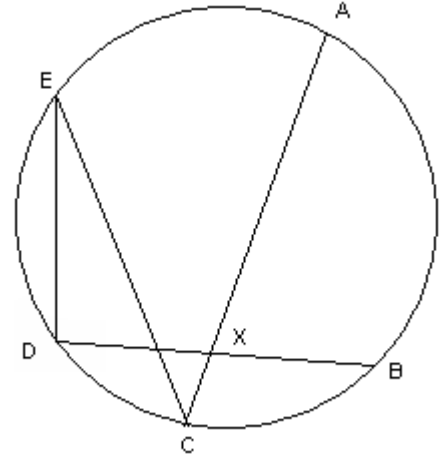


3. \overline{CD} is internally tangent to both circles. \overline{EF} is externally tangent to both circles. A and B are the centers of the circles. If $AP = 25$, $PB = 10$ and $CD = 28$, find the exact length of \overline{EF} in simplest form or rounded to the nearest hundredth.



Circles and Spheres
March 2001

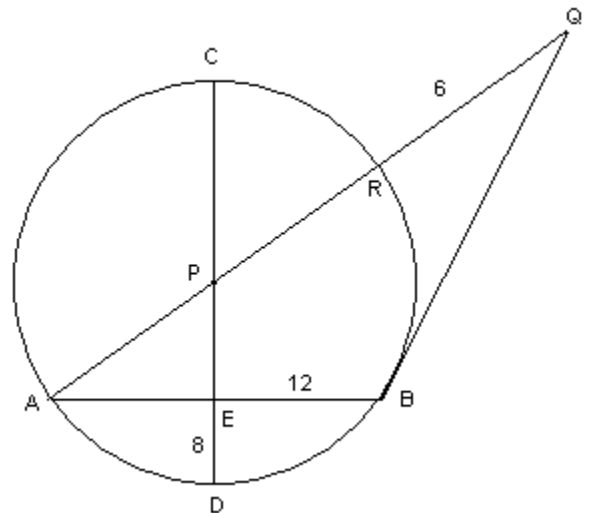
1. Given: $m\text{Arc}AB = 110^\circ$ and $m\angle AXB = 80^\circ$, find $m\angle DEC$.
 Express your answer in degrees.



2. A 6" radius spherical bowling ball has a hollow, concentric interior with radius of 2". Inside the interior is a spherical marble with radius $\frac{3}{8}$ ". As the bowling ball rolls, the marble also rolls so that it is always at the bottom of the interior. How many revolutions does the marble make while the bowling ball makes 60 revolutions?

3. Given: Center P, $\overline{CD} \perp \overline{AB}$,
 \overline{QB} is tangent at B,
 $DE = 8$, $BE = 12$,
 $QR = 6$.

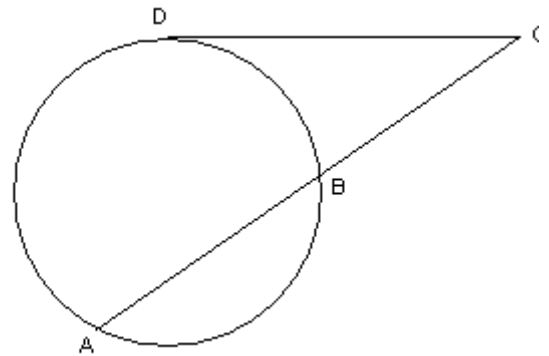
Find BQ . Express in simplest radical form.



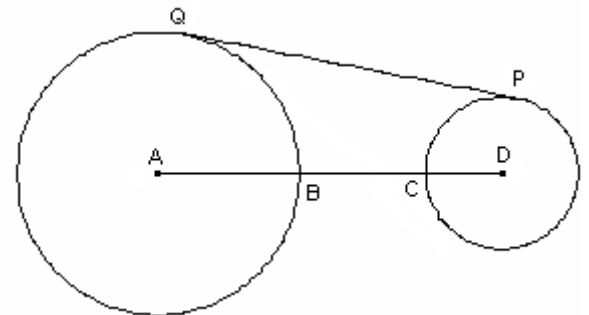
Circles and Spheres

March 2002

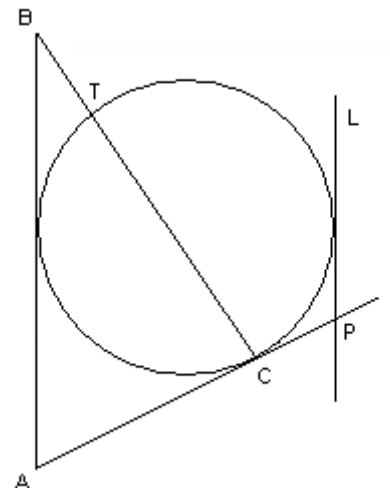
1. Find the length of the tangent segment CD , if $AB = 7$ and $BC = 9$.



2. A and D are centers. $AB = 12$, $BC = 8$ and $CD = 5$.
Find the length of tangent PQ .



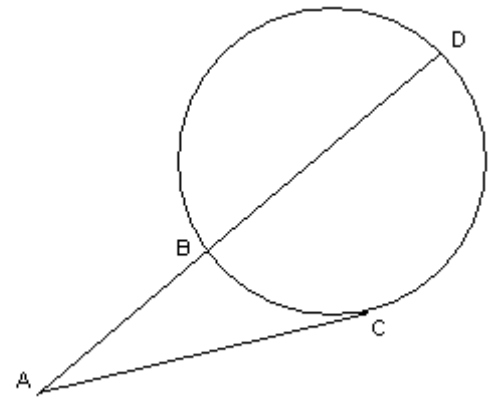
3. The circle at right is tangent to the two parallel lines AB and PL . \overline{AP} is tangent to the circle at C . $\overline{BC} \perp \overline{AP}$. If $AC = 15$, $BC = 20$, and $AB = 25$.
Find the length of segment \overline{BT} .



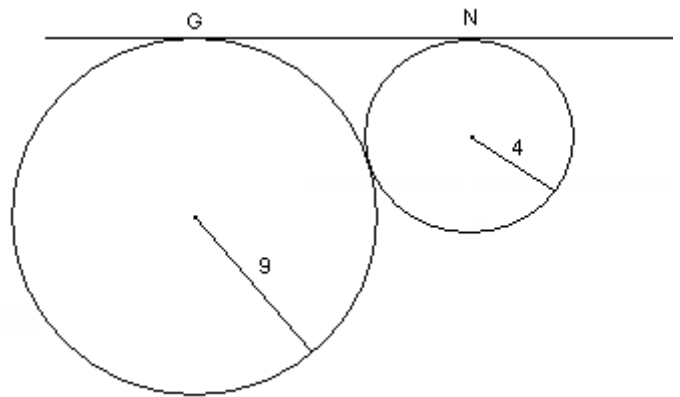
Circles and Spheres

March 2003

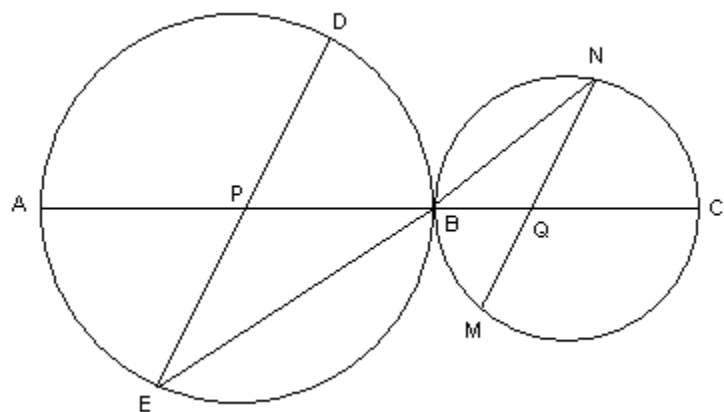
- Given: $m\text{Arc}BC = 68^\circ$, tangent segment AC ,
 $m\text{Arc}CD = m\text{Arc}BD$
 Find $m\angle A$.



- Circles P and L are tangent and have radii of 9 and 4 respectively. Find the length of the common tangent GN .



- The tangent circles meet at B . $AB = 32$, $BC = 24$, $PD = QN = 12$, $PE = 16$, $AP > PB$, and $\overline{DE} \parallel \overline{MN}$. Find the length of segment MQ .

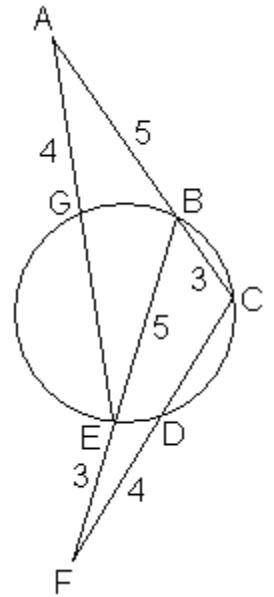


Circles and Spheres(No Calculators)

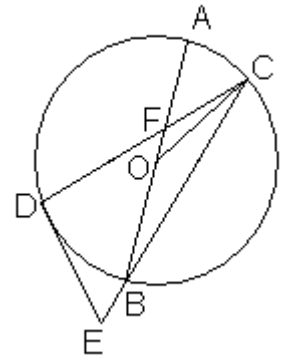
March 2005

1. If the radius of the earth is 4000 miles at the equator, how many more miles does it take to circle the earth at the equator at an altitude of 1 mile than it does at the surface. Express your answer in terms of π .

2. 2 secants are drawn to the circle shown from points A and F . Points B , C , D , E , and G are on the circle, and are also on the secants. 6 distances between points are shown. Find the perimeter of quadrilateral $ACFE$.



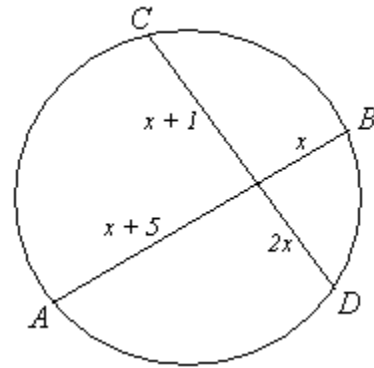
3. In circle O , segment AB is a diameter, segment CE passes through B , segment DE is tangent to the circle at D , $m\angle AOC = 30^\circ$, and $m\angle CED = 50^\circ$. Find the $m\angle AFD$.



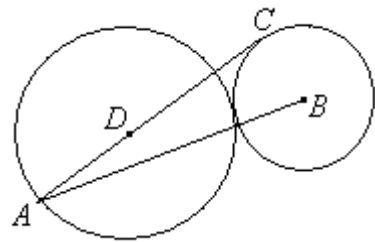
Circles and Spheres(No Calculators)

March 2006

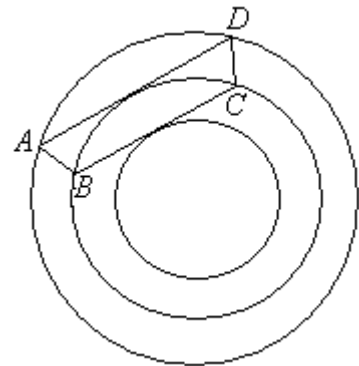
1. Given the circle as marked, find the numerical length of chord AB .



2. The circles have radii of 8 and 5. Tangent AC passes through the center D of the larger circle and is tangent to the smaller circle at C . Find the length of segment AB , where B is the center of the smaller circle.



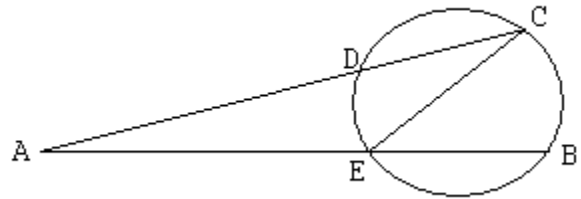
3. The radii of the three concentric circles are 6, 12, and 18. Tangents segments BC and AD have points of tangency which are collinear with their center. Find the length of segment DC in simplest and exact form.



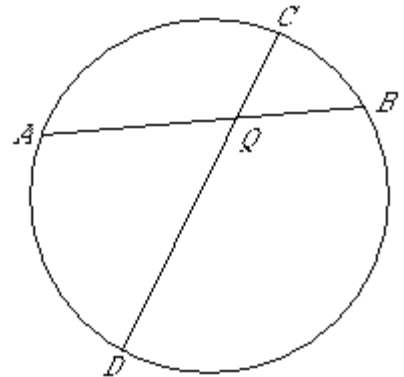
Circles and Spheres(No Calculators)

March 2008

- In the figure at right, $m\angle A = 25^\circ$, $m\angle DE = 50^\circ$.
Find the degree measure of $\angle CEB$.



- \overline{AB} and \overline{CD} are chords of the circle. \overline{CD} bisects \overline{AB} at Q . AQ is 3 units longer than CQ and 4 units shorter than DQ . Find the length of segment CD .



- Chord \overline{BC} of the larger circle below is the perpendicular bisector of chord AD at P , the center of the smaller circle. E is the endpoint of the diameter of the smaller circle and the center of the larger circle. Find the ratio of AD to BE .

