

MAINE ASSOCIATION OF MATH LEAGUES

RULES GOVERNING QUESTIONS, ANSWERS, AND GRADING

2015-2016

Introduction

Philosophy. It is the intent of MAML to promote Maine high school mathematics competitions. The questions in these competitions should be fair, appropriate and clear, and they should challenge students at all grade and ability levels.

Ground Rules, Goals, and Assumptions. In sponsoring competitions and creating the questions students will answer, the following ground rules, goals, and assumptions shall be paramount:

1. Each question shall have a clear, unique answer so that the students' responses can, without ambiguity, be marked either correct, for full credit, or incorrect, for no credit.
2. Each question shall require the student to write a short answer in a designated space.
3. Each question shall be drawn from the material covered in the standard pre-calculus high school curricula.
4. Unless otherwise stated, either directly or through context, all variables and constants in questions and answers shall be assumed to be real numbers and the term *number* shall mean a real number. When a question requests a *value*, it shall be requesting a number and not any expression or equation containing a variable.
5. If the answer to a question involves units of any type, the question itself shall specify the units desired. The units do not need to be repeated in the answer.
6. All numbers in questions and answers shall be in base 10 unless otherwise stated in the question.
7. All polygons shall be named with upper-case English letters, one to a vertex, and the letters may be written in either clockwise or counter-clockwise order around the polygon. Each letter may refer to the vertex itself, the interior angle at the vertex, or the measure of this angle. All polygons shall be non-degenerate. In the case of triangles, the side opposite each vertex shall be named with the lower-case counterpart to the upper-case letter used to label the vertex. These lower-case letters can be used to refer either to the referent side, or to its length.
8. Student competitors shall be assumed to have a reasonable facility with the English language, a thorough knowledge of high school mathematical terms and symbols, and a reasonable knowledge of non-mathematical information appropriate for a high school student—particularly of topics that lend themselves easily to mathematics.
9. Students shall be assumed to have sufficient clarity in their penmanship to allow graders who are not their teachers to easily discern the intent of their written answers.
10. Students may use a calculator only on specified rounds of questions. Calculators are not allowed at the State Meet.

Forms of Answers

To promote the use of standard mathematical terms, symbols, and syntax vital to all mathematical communication and to ensure the correct and efficient grading of all student work, it is imperative that appropriate rules be applied in establishing acceptable forms for students' answers to questions. The overwhelming majority of these rules are evident in every high school mathematics textbook and every mathematics classroom in Maine. Some of these are repeated here, together with clarifications and additional rules deemed necessary to the successful operation of the competitions. All rules after the first three are arranged by the category of the answer (NUMBERS, EXPRESSIONS, EQUATIONS AND INEQUALITIES, and OTHER ANSWERS). Each rule is numbered for reference in the Quick Reference Guide.

1. Answer the Question! The student will receive credit only for a complete and correct answer to the actual question asked. If an answer is correct but contains extraneous information such as unnecessary units or data not requested in the question, the student's answer will be marked as correct only if all information provided is correct.

2. Simplify! There are an infinite number of ways to write most answers and graders must work quickly and accurately. Therefore, all answers will be marked as correct only if they are in fully simplified form, as defined by common mathematical practice augmented by the specific rules that follow.

3. Unless Otherwise Specified! When the directives in a particular question contradict any of the following rules for forms of answers, the contradicted rule will be waived and the specific directives of the question will take precedence.

NUMBERS. . . .

All Numbers

4. All operations must be performed before an answer is written.
5. An answer must be exact and not rounded (non-calculator questions).
6. An answer must be either exact or rounded to four decimal places (calculator questions). Answers must be rounded according to the 5/4 rounding method. Answers involving currency must be exact or rounded to 2 decimal places.
7. If an answer is requested in a base other than 10 and the question explicitly says, "Find in base b , . . .", the base designation may be omitted in the answer. If the question is not so explicit, the student must indicate the base using one of the three accepted methods: N_b , $(N)_b$, or $N_{\text{the word for the number } b \text{ spelled out}}$.

Natural Numbers

8. The set of natural numbers, also known as counting numbers is $\{1, 2, 3, \dots\}$. Note that it does not include 0.

Rationals

9. All fractions must be fully reduced. Improper fractions and mixed numbers are both acceptable.

Reals

10. Real number and extended real number answers can employ the symbols π , e , and ∞ .
11. Radical expressions must be fully simplified. Specifically:
- a) they can have no radical in a denominator and no fraction within a radical.
 - b) an integer radicand that is taken to the n^{th} root must have no integer factor > 1 that is an n^{th} power.
 - c) radicals must be expressed using the least possible index.
 - d) if rational exponents are allowed, the base must be the smallest possible integer.
 - e) $\sqrt{x^2} = |x|$

Complex Numbers

12. The term i is mandatory for all imaginary and complex number answers.
13. Complex numbers must be written in one of the following forms:
 $a + bi$, $bi + a$, $\frac{a + bi}{c}$, or $\frac{bi + a}{c}$. In the latter two, c must not equal 1 and, if a , b , and c are all integers, they must be relatively prime. It is also acceptable to replace the b with the number 0 when the question requests a complex number and the actual answer has no imaginary component. In addition, if either a or b is not an integer it is permitted to factor the answer into a product of a real number and a complex number providing the complex factor meets the above requirements. Complex numbers cannot be written in polar form or using *cis* notation.

Ordered n-tuples

14. All ordered pairs, ordered triples, and ordered n -tuples must be written inside parenthesis, in the prescribed order, and with commas separating the listed terms.

Probabilities

15. All probabilities must be written as fractions, as decimal numbers, or as percentages.

Ratios

16. Ratios should be written as the fraction $\frac{A}{B}$. Also acceptable are one of the following forms: $A:B$, or A to B . In all three cases, the A and B must be in such form that the fraction $\frac{A}{B}$ meets the radical simplification and fraction reduction rules. In the event that B is 1, the ratio must be written with a 1 in the denominator, such as $\frac{\pi}{1}$.

Angles

17. Because units in answers are unnecessary, angle measures in answers must be in the same units, degrees or radians, as used in the question.

NUMBERS continued. . . .

Scientific Notation 18. Scientific notation will not be accepted in an answer.

Taboo Symbols 19. Calculator-specific, computer programming-specific, and mathematical analysis-specific notations, when they differ from standard and accepted high school mathematical notations, are not permissible. In particular, the use of the calculator symbol “E” for scientific notation, a caret (“^”) to indicate an exponent, and an ordered pair to indicate a complex number are all not allowed.

Negative Exponents 20. Negative exponents are not acceptable in denominators.

EXPRESSIONS. . . .

General 21. All rules for NUMBERS listed above apply.

22. All like terms with rational coefficients must be combined.

Polynomials 23. In a polynomial expression, either all operations must be carried out or the expression must be written in fully factored form.

Symbols 24. Variables and other place-holder symbols from a question that are to be used in an answer must be written identically

EQUATIONS AND INEQUALITIES. . . .

General 25. All rules for NUMBERS and EXPRESSIONS listed above apply.

Equations 26. If all the coefficients and the constant in an equation are integers, they must be relatively prime.

Established Forms 27. If an answer contains the equation of a geometric feature that has a standard form, such as a line, circle, ellipse, parabola, or hyperbola, the equation may optionally include the number 0 as a place-holder for any constant or coefficient in order to preserve the sanctity of the form.

Multiple Equations 28. Answers involving multiple equations may be written in any order. Commas may be used as separators. In addition, if the equations involved are solutions for two or more variables given in the question, the student may choose to answer with an ordered n -tuple of values, in which case the elements will be considered to be the respective variable solutions in alphabetical order.

EQUATIONS AND INEQUALITIES continued. . . .

Compound Inequalities

29. Compound inequality answers involving a variable and two inequality symbols must be written so that the expression between the two inequality symbols is simply the variable itself appearing as a single term without coefficient.

Multiple Inequalities

30. When multiple inequalities or intervals are used in an answer, the inequalities and intervals must be separated either by one of the logical operators “AND” or “OR” or by one of the set notation symbols “ \cap ” or “ \cup ”. Commas will not be accepted in these answers. The only accepted use of a comma in the context of an inequality is to separate an inequality from an exception involving the \neq symbol. In this case, the comma will signify the word “except”.

Set Notation

31. Set notation, when appropriate, will be acceptable. This includes the use of {braces} and the symbols \in and \emptyset .

Domain, Range, and Intervals

32. Interval notation, set-builder notation, and inequalities are all acceptable ways to describe continuous values. In addition, the terms “ALL REALS”, “ALL REALS EXCEPT. . .”, and “ALL REALS BUT. . .” may be used.

Graphs

33. Graphs will not be accepted in answers.

OTHER ANSWERS. . . .

Lists

34. When an answer is a list, the list can be presented in any clear form, including using commas as separators. In this context, each comma will signify the word “OR”.

Matrices and Determinants

35. The elements of any matrix or determinant must be written in the correct positions. A matrix must be enclosed in [brackets] or (parentheses). A determinant must be enclosed in | vertical lines |. Calculator-specific notation will not be accepted for matrices.

Points on a Coordinate Plane

36. If an answer is a point on a coordinate plane, then it must be shown as an ordered pair (a,b), including the parentheses with the x value first and the y value second, or written as $x = a$, $y = b$.

Conic Sections

37. In the format $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ (or similar for other sections), the a and b may be left as a^2 , b^2 or simplified.

MAML Quick Reference Guide

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Integer	4		4.0 [1] 4.0000 (even on calculator round) [1] $3.\bar{9}$ [1,2] $8/2$ [1,2,4,9] $\begin{array}{r l} 2 & 3 \\ 2 & 5 \end{array}$ [1] $2 \csc 30^\circ$ [1] 400% [1] 4.0×10^0 [1,18] $9 - 5$ [1,2,4] 2^2 [1,19] 4.0 E 0 [1,19] 100_2 [1] – other bases must be requested
Value	$1\frac{1}{7}$	$\frac{8}{7}$ $\overline{1.142857}$ 1.1429 (calc only)	1.142857 [5] $\frac{104}{91}$ [2,9] 1.1429 (non-calc only) [5]
Value	$\frac{2}{3}$	$\bar{.6}$.6667 (calc only)	$\bar{.66}$ (or any number of 6's >1) [9] $\bar{.66}$ [9] .6667 (non-calc only) [5] .67 (calc only) [6] .6666 (calc only) [6]
Value - currency and the answer before rounding is 6.2450	\$6.25	6.25	6.2450 (calc only) [6] 6.24 [6] \$6.24 [6]
Number	$\frac{\sqrt{6}}{2}$	1.2247 (calc only)	$\frac{\sqrt{3}}{\sqrt{2}}$ [11] $\sqrt{\frac{3}{2}}$ [11] $\frac{\sqrt{6} - \sqrt{3}}{2 - \sqrt{3}}$ [11]

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Decimal Number rounded to 2 decimal points and the answer before rounding is $1\frac{2}{7}$	1.29		$1\frac{29}{100}$ [1] $1\frac{2}{7}$ [1] $\frac{9}{7}$ [1] 1.2900 or 1.2857 (even on calc round) [1]
Number in base 4	20_4	$20)_4$ 20_{four} 20	8 [1]
Real	π	3.1416 (calc only)	3.14 [5] 3.1415926535898 [5] $\frac{22}{7}$ [5]
Real	$\sqrt{3}$	1.7321 (calc only) $3^{\frac{1}{2}}$	$\frac{3}{\sqrt{3}}$ [11a] $\sqrt[4]{9}$ [11c] $9^{\frac{1}{4}}$ [11d]
Real	$4(\sqrt[3]{2})$	$4 \cdot 2^{\frac{1}{3}}$ $2^{\frac{7}{3}}$ 5.0397 (calc only)	$\sqrt[3]{128}$ [11b] $\frac{4}{2^{\frac{-1}{3}}}$ [20] $4^{\frac{7}{6}}$ [11d] $2^{\frac{14}{6}}$ [9]
Radical Expression and the answer is $\sqrt{x^2y^3z^4}$	$ x yz^2\sqrt{y}$		$ xy z^2\sqrt{y}$ [11e] $xyz^2\sqrt{y}$ [11e]
Radical Expression and the answer is $\sqrt{(-3x)^2}$	$3 x $		$-3x$ [11e] $ -3x $ [11e]
Imaginary Complex	$2+4i$	$4i+2$	$\frac{2+4i}{1}$ or $\frac{4i+2}{1}$ [13] (1,2) [19] $2(1+2i)$ or $2(2i+1)$ [4] $2\sqrt{5}(\cos 63.4349^\circ + i \sin 63.4349^\circ)$ [13] $2\sqrt{5}(\text{cis } 63.4349^\circ)$ [13]

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Complex	$\frac{15}{2} + \frac{5i}{2}$	$\frac{5i}{2} + \frac{15}{2}$ $\frac{15+5i}{2}$ or $\frac{5i+15}{2}$	$\frac{1}{2}(15+5i)$ or $\frac{1}{2}(5i+15)$ [4] $\frac{5}{2}(3+i)$ or $\frac{5}{2}(i+3)$ [4] $\frac{5(i+3)}{2}$ or $\frac{5(3+i)}{2}$ [4]
Complex	2	2+0i or 0i+2	
Ordered Pair	(7,-2)		7,-2 [14] (-2,7) [14]
Probability	$\frac{\sqrt{3}}{2}$	50√3% .8660 or 86.6025% (calc only)	√3 : 2 [15] .8660 or 86.6025% (non-calc only) [5]
Probability	$\frac{7}{8}$	87.5% 0.875 0.8750 (calc only)	7:8 or 7 to 8 [15] 7:1 or 7 to 1 [15]
Ratio	$\frac{5}{2}$	5 to 2 5 : 2	$2\frac{1}{2}$ or 2.5 [16] 10 : 4 [16] $\frac{2.5}{1}$ or $2\frac{1}{2}:1$ [16]
Ratio	$\frac{\sqrt{10}}{1}$	√10 : 1 √10 to 1	√10 [16] $2\sqrt{5} : \sqrt{2}$ or $2\sqrt{5}$ to $\sqrt{2}$ [16]
Angle in the range $0^\circ \leq \theta < 360^\circ$	300°	300	-60° [1] $\frac{5\pi}{3}$ [17] $-\frac{\pi}{3}$ [1,17]
Angle in the range $0 \leq \theta < \frac{\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{6}$ radians 0.5236 (calc only)	30° [17] $-\frac{11\pi}{6}$ [1]
Algebraic Expression	4x - 7y + 2	{any of the 5 other possible orders of the 3 terms}	2x + 2 + y + 2x - 8y [22]
Algebraic Expression	$5\sqrt{3}x + 8$	$8 + 5\sqrt{3}x$	$\sqrt{3}x + 8 + 4\sqrt{3}x$ [22] ($\sqrt{3}x$ is the term)

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Algebraic Expression	$2x^3 + 10x^2 + 12x$	$2x(x+3)(x+2)$ or any of the other orders of these three factors	$x(2x^2 + 10x + 12)$ [23]
Algebraic Expression	$2a^2b^2 - 2a^2c^2 + c^4 - b^4$	$(b+c)(b-c)$ $(2a^2 - b^2 - c^2)$	$(b+c)(b-c)((a+b)(a-b) + (a+c)(a-c))$ [23]
Binomial Expansion of $(a+b)^5$	$10a^3b^2$ (3rd term)		$10a^2b^3$ [term number based on descending order of first variable]
Factored Expression	$(c+3)(c-2)$	$(c-2)(c+3)$	$(x+3)(x-2)$ [24]
Solution for y	$y = 7x^2 - 6$	$y = -6 + 7x^2$ $7x^2 - 6 = y$ $-6 + 7x^2 = y$	$7x^2 = y + 6$ [1]
Equation	$3a + 5 = 2b$	{any of the 47 other equivalent, simplified forms, including using a 0 term}	$39a + 65 = 26b$ [26]
Equation in specified form	$x^2 + y^2 = 1$	$(x+0)^2 + (y+0)^2 = 1$	
System solution for a, b, and c	$a = 2, b = -1, c = 3$	{any of the 6 orders of the 3 equations, with or without commas} $(2, -1, 3)$	$2, -1, 3$ [1,28]
Domain for x	$0 < x \leq 2$	$x > 0$ and $x \leq 2$ $x > 0 \cap x \leq 2$ { $x : x > 0$ and $x \leq 2$ } { $x : 0 < x \leq 2$ } { $x : x > 0 \cap x \leq 2$ } --all 6 of these in either order $(0, 2]$	$1 < (x+1) \leq 3$ [29] $0 < 3x \leq 6$ [29] $x > 0$ or $x \leq 2$ [1] $x > 0 \cup x \leq 2$ [1] $x > 0, x \leq 2$ [30] $[2, 0)$

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Domain for x	$x > 5$ or $x \leq -3$	$x > 5 \cup x \leq -3$ $\{x : x > 5 \text{ or } x \leq -3\}$ $\{x : x > 5 \cup x \leq -3\}$ $(-\infty, -3] \cup (5, \infty)$ --all five in either order	$(-\infty, -3] \cap (5, \infty)$ [1] $x > 5$ and $x \leq -3$ [1]
Range of $f(x)$	$f(x) < 2$, except $f(x) \neq 0$	$f(x) < 2$, but $f(x) \neq 0$ $f(x) < 2, f(x) \neq 0$ $f(x) < 2$ and $f(x) \neq 0$ $(-\infty, 0) \cup (0, 2)$ all reals < 2 , except 0 $y < 2, y \neq 0$	$f(x) < 2$ or $f(x) \neq 0$ [1] $(-\infty, 0) \cap (0, 2)$ [1] $f < 2$, except $f \neq 0$ [1]
Solution such that there are multiple solutions for a single variable	$x = 0$ or $x = 1$ or $x = 5$	$x = 0, 1, 5$ $x \in \{0, 1, 5\}$ $x = 0, x = 1, x = 5$ $x = 0$ and 1 and 5	$x = \{0, 1, 5\}$ [1] – x is not a set a graph on the number line [33]
Matrix	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$	$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$	$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ [35] $[[1 \ 2] \ [3 \ 4]]$ [35]
Point on a coordinate plane	$(5, -3)$	$x = 5, y = -3$	$5, -3$ [36] $(-3, 5)$ [36]
Conic Sections	$\frac{(x-3)^2}{16} +$ $\frac{(y+2)^2}{25} = 1$	$\frac{(x-3)^2}{4^2} +$ $\frac{(y+2)^2}{5^2} = 1$	