MAINE ASSOCIATION OF MATH LEAGUES
DEFINITIONS 2017 - 2018

PLEASE NOTE THAT THERE IS AN ACCOMPANYING DOCUMENT AVAILABLE THAT DEFINES THE EXPECTATIONS FOR ALL THREE POINT PROBLEMS IN EACH CATEGORY.

ABSOLUTE VALUES/INEQUALITIES: The questions will be based on any inequality or absolute value problem up to a third degree equation or inequality in factorable form.

ALGEBRAIC FRACTIONS AND FACTORING: Questions will be geared to techniques of factoring and the operations on or simplification of algebraic fractions, or solving algebraic fractional equations. These questions may also employ LCM's, GCF's or LCD's.

ARITHMETIC: There will be 2 questions in each round dealing with the specialty categories. The remaining question will be arithmetic in nature, either simple number theory, consumer based, or pre Algebra in nature.
Meet 1: "*" Star Operations
Meet 2: Ratio/Proportionality
Meet 3: Percentage
Meet 4: Literal Equations not to exceed the Algebra 1 level except on the team round.
Meet 5: Statistics **

** The Statistics questions will be geared to range, mean, mode, median, histograms or relative frequency histograms. Range is defined to be the difference between the largest and smallest numbers in the data set. In addition, the problem writer has the flexibility to give only one statistics question, instead of two like the other meets. If standard deviation appears in a question, it will always be a sample standard deviation. The problem writer will only include standard deviation on the team round.

AREAS/VOLUMES: The questions will be based on the areas of plane geometric figures, working with total and lateral areas of solids and finding the volumes of cylinders, cones, pyramids, frustums, prisms or spheres.

CIRCLES/SPHERES: The questions will be based on any non-coordinate circle or sphere concept specifically dealing with angles related to a circle, radii, chords, secants, tangents, angle-arcs, circumference or arc length.

COMPLEX NUMBERS: The questions will be based on the operations of complex numbers including simplifying complex number expressions, absolute value, roots of complex numbers, modulus, reciprocals, and roots of polynomial functions with real or complex coefficients. DeMoive's Theorem will only appear in the Team Round.

CONICS: This category will deal with the properties of circles, ellipses, parabolas, or hyperbolas as applied to the coordinate plane. At most one latus rectum question. No trigonometric equations will be used.

COUNTING PRINCIPLES AND BINOMIAL THEOREM: The questions will involve combinations, permutations, binomial theorem and other basic counting principles. There will be at least one Binomial Theorem problem. Problems will not include trinomials except on the team round.

EXPONENTS/RADICALS: The questions will be based on the operations of exponents/radicals to include relationships, simplifications or equations.

FUNCTIONS: The questions will be based on the operations of functions, function notation, domain-range relationships, composition, and inverses. This category will exclude trigonometric functions.

GEOMETRIC SIMILARITIES: Questions will deal with Geometric Similarities and proportionality of polygons or solids.
LINEAR COORDINATE GEOMETRY: The questions in this category will involve the equations of the coordinate plane including parallels and perpendiculars. It also may involve other geometric figures such as triangles, circles or quadrilaterals as they apply to the coordinate axes. If the answer to any problem is an equation of a line, the problem writer will specify (by example, not name) the form of the equation to be used.

LINES. ANGLES AND POLYGONS: The questions will deal with any line or angle relationship in a plane or any polygon material from triangles through n-gons. The questions will not deal with area or questions better suited for circles or spheres.

LOGS/LOG EQUATIONS: The questions will deal with concepts of logs, log equations and applications of logarithms.

MATRICES, DETERMINANTS AND SYSTEMS OF EQUATIONS: The questions will be based on simplification of determinants not to exceed a 4 by 4, matrix operations, matrix equations, inverses not to exceed a 3 by 3, transpositions, and applications of matrix and determinant properties.

NUMBER THEORY: The questions will be based on the theory of natural numbers to include but not limited to least common multiples, greatest common factors or divisors, divisibility tests, number of divisors, sum of divisors, primes and composites, remainders, bases and modula mathematics. It should be noted that despite what apparently some textbooks report, zero is NOT a natural number.

POLYNOMIALS: The questions will be based on the operations of polynomial expressions, zeros of functions, solving polynomial equations, applications of the theory of equations. Complex roots will be allowed in 5-point questions and the team round.

PROBABILITY: The questions will involve concepts of combinations, permutations, expected value problems, tree diagrams, mutually exclusive, complementary, independent and dependent events as applied to probability. This category will also include conditional probability and Binomial Theorem as applied to probability.

SERIES AND SEQUENCES: Questions will be based on arithmetic and geometric sequences and series and their related concepts.

TRIGONOMETRIC MECHANICS: The questions will be based on right and non-right triangles and their applications, circular trigonometry, and radian measure. One question may deal with the Laws of Sines or Cosines to non-right triangles. The Team Round may have a question dealing with angular and linear velocity.

TRIGONOMETRIC EQUATIONS AND IDENTITIES: The questions will deal with solving trigono metric equations with a specified domain, simplifying and working with trigonometric identities, as well as formulas for double, half, sum and difference. The team round may involve trig equations with unrestricted domains. In this round, Arcsin and Sin-1 denote functions, where arcsin and sin-1 denote relations. In addition, students should understand the difference between Sin x and sin x. Similar expectations will hold for the other trigonometric functions.

MAINE ASSOCIATION OF MATH LEAGUES
RULES GOVERNING QUESTIONS, ANSWERS, AND GRADING
2017-2018

Introduction

Philosophy. It is the intent of MAML to promote Maine high school mathematics competitions. The questions in these competitions should be fair, appropriate and clear, and they should challenge students at all grade and ability levels.

Ground Rules, Goals, and Assumptions. In sponsoring competitions and creating the questions students will answer, the following ground rules, goals, and assumptions shall be paramount:

1. Each question shall have a clear, unique answer so that the students' responses can, without ambiguity, be marked either correct, for full credit, or incorrect, for no credit.

2. Each question shall require the student to write a short answer in a designated space.

3. Each question shall be drawn from the material covered in the standard pre-calculus high school curricula.

4. Unless otherwise stated, either directly or through context, all variables and constants in questions and answers shall be assumed to be real numbers and the term *number* shall mean a real number. When a question requests a *value*, it shall be requesting a number and not any expression or equation containing a variable.

5. If the answer to a question involves units of any type, the question itself shall specify the units desired. The units do not need to be repeated in the answer.

6. All numbers in questions and answers shall be in base 10 unless otherwise stated in the question.

7. All polygons shall be named with upper-case English letters, one to a vertex, and the letters may be written in either clockwise or counter-clockwise order around the polygon. Each letter may refer to the vertex itself, the interior angle at the vertex, or the measure of this angle. All polygons shall be non-degenerate. In the case of triangles, the side opposite each vertex shall be named with the lower-case counterpart to the upper-case letter used to label the vertex. These lower-case letters can be used to refer either to the referent side, or to its length.

8. Student competitors shall be assumed to have a reasonable facility with the English language, a thorough knowledge of high school mathematical terms and symbols, and a reasonable knowledge of non-mathematical information appropriate for a high school student—particularly of topics that lend themselves easily to mathematics.

9. Students shall be assumed to have sufficient clarity in their penmanship to allow graders who are not their teachers to easily discern the intent of their written answers.

10. Students may use a calculator only on specified rounds of questions. Calculators are not allowed at the State Meet.

Forms of Answers

To promote the use of standard mathematical terms, symbols, and syntax vital to all mathematical communication and to ensure the correct and efficient grading of all student work, it is imperative that appropriate rules be applied in establishing acceptable forms for students' answers to questions. The overwhelming majority of these rules are evident in every high school mathematics textbook and every mathematics classroom in Maine. Some of these are repeated here, together with clarifications and additional rules deemed necessary to the successful operation of the competitions. All rules after the first three are arranged by the category of the answer (NUMBERS, EXPRESSIONS, EQUATIONS AND INEQUALITIES, and OTHER ANSWERS). Each rule is numbered for reference in the Quick Reference Guide.

1. Answer the Question! The student will receive credit only for a complete and correct answer to the actual question asked. If an answer is correct but contains extraneous information such as unnecessary units or data not requested in the question, the student's answer will be marked as correct only if all information provided is correct.

2. Simplify! There are an infinite number of ways to write most answers and graders must work quickly and accurately. Therefore, all answers will be marked as correct only if they are in fully simplified form, as defined by common mathematical practice augmented by the specific rules that follow.

3. Unless Otherwise Specified! When the directives in a particular question contradict any of the following rules for forms of answers, the contradicted rule will be waived and the specific directives of the question will take precedence.

NUMBERS. . . .

All Numbers

4. All operations must be performed before an answer is written.

5. An answer must be exact and not rounded (non-calculator questions).

6. An answer must be either exact or rounded to four decimal places (calculator questions). Answers must be rounded according to the 5/4 rounding method. Answers involving currency must be exact or rounded to 2 decimal places.

7. If an answer is requested in a base other than 10 and the question explicitly says, "Find in base *b*, . . .", the base designation may be omitted in the answer. If the question is not so explicit, the student must indicate the base using one of the three accepted methods: N_b, $N)_b$, or $N_{\text{the word for the number b spelled out}}$.

Natural Numbers

8. The set of natural numbers, also known as counting numbers is $\{1, 2, 3, \ldots\}$. Note that it does not include 0.

Rationals

9. All fractions must be fully reduced. Improper fractions and mixed numbers are both acceptable.

Reals

10. Real number and extended real number answers can employ the symbols π, e, and ∞.

11. Radical expressions must be fully simplified. Specifically:

a) they can have no radical in a denominator and no fraction within a radical.

b) an integer radicand that is taken to the n^{th} root must have no integer factor > 1 that is an n^{th} power.

c) radicals must be expressed using the least possible index.

d) if rational exponents are allowed, the base must be the smallest possible integer.

e) $\sqrt{x^2} = |x|$

Complex Numbers

12. The term *i* is mandatory for all imaginary and complex number answers.

13. Complex numbers must be written in one of the following forms:

$a + bi$, $bi + a$, $\frac{a + bi}{c}$, or $\frac{bi + a}{c}$. In the latter two, c must not equal 1 and, if a, *b*, and *c* are all integers, they must be relatively prime. It is also acceptable to replace the *b* with the number 0 when the question requests a complex number and the actual answer has no imaginary component. In addition, if either *a* or *b* is not an integer it is permitted to factor the answer into a product of a real number and a complex number providing the complex factor meets the above requirements. Complex numbers cannot be written in polar form or using *cis* notation.

Ordered n-tuples

14. All ordered pairs, ordered triples, and ordered *n*-tuples must be written inside parenthesis, in the prescribed order, and with commas separating the listed terms.

Probabilities

15. All probabilities must be written as fractions, as decimal numbers, or as percentages.

Ratios

16. Ratios should be written as the fraction $\frac{A}{B}$. Also acceptable are one of the following forms: *A*:*B*, or *A* to *B*. In all three cases, the *A* and *B* must be in such form that the fraction $\frac{A}{B}$ meets the radical simplification and fraction reduction rules. In the event that *B* is 1, the ratio must be written with a 1 in the denominator, such as $\frac{\pi}{1}$.

Angles

17. Because units in answers are unnecessary, angle measures in answers must be in the same units, degrees or radians, as used in the question.

NUMBERS continued. . . .

Scientific Notation **18.** Scientific notation will not be accepted in an answer.

Taboo Symbols **19.** Calculator-specific, computer programming-specific, and mathematical analysis-specific notations, when they differ from standard and accepted high school mathematical notations, are not permissible. In particular, the use of the calculator symbol "E" for scientific notation, a caret (" ^ ") to indicate an exponent, and an ordered pair to indicate a complex number are all not allowed.

Negative Exponents **20.** Negative exponents are not acceptable in denominators.

EXPRESSIONS. . . .

General **21.** All rules for NUMBERS listed above apply.

22. All like terms with rational coefficients must be combined.

Polynomials **23.** In a polynomial expression, either all operations must be carried out or the expression must be written in fully factored form.

Symbols **24.** Variables and other place-holder symbols from a question that are to be used in an answer must be written identically

EQUATIONS AND INEQUALITIES. . . .

General **25.** All rules for NUMBERS and EXPRESSIONS listed above apply.

Equations **26.** If all the coefficients and the constant in an equation are integers, they must be relatively prime.

Established Forms **27.** If an answer contains the equation of a geometric feature that has a standard form, such as a line, circle, ellipse, parabola, or hyperbola, the equation may optionally include the number 0 as a place-holder for any constant or coefficient in order to preserve the sanctity of the form.

Multiple Equations **28.** Answers involving multiple equations may be written in any order. Commas may be used as separators. In addition, if the equations involved are solutions for two or more variables given in the question, the student may choose to answer with an ordered *n*-tuple of values, in which case the elements will be considered to be the respective variable solutions in alphabetical order.

EQUATIONS AND INEQUALITIES continued. . . .

Compound Inequalities

29. Compound inequality answers involving a variable and two inequality symbols must be written so that the expression between the two inequality symbols is simply the variable itself appearing as a single term without coefficient.

Multiple Inequalities

30. When multiple inequalities are used in an answer, the inequalities must be separated by one of the logical operators "AND" or "OR". If multiple intervals or sets are used, they must be separated by one of the set notation symbols $\cap$ **or** $\cup$**.** Commas will not be accepted in these answers. The only accepted use of a comma in the context of an inequality is to separate an inequality from an exception involving the $\neq$ symbol. In this case, the comma will signify the word "except".

Set Notation

31. Set notation, when appropriate, will be acceptable. This includes the use of {braces} and the symbols $\in$ and $\varnothing$**. Mixing set notation and compound inequality notation is not acceptable.**

Domain, Range, and Intervals

32. Interval notation, set-builder notation, and inequalities are all acceptable ways to describe continuous values. In addition, the terms "ALL REALS", "ALL REALS EXCEPT. . .", and "ALL REALS BUT. . ." may be used.

Graphs

33. Graphs will not be accepted in answers.

OTHER ANSWERS. . . .

Lists

34. When an answer is a list, the list can be presented in any clear form, including using commas as separators. In this context, each comma will signify the word "OR".

Matrices and Determinants

35. The elements of any matrix or determinant must be written in the correct positions. A matrix must be enclosed in [brackets] or (parentheses). A determinant must be enclosed in $\left|$ vertical lines $\right|$. Calculator-specific notation will not be accepted for matrices.

Points on a Coordinate Plane

36. If an answer is a point on a coordinate plane, then it must be shown as an ordered pair (a,b), including the parentheses with the x value first and the y value second, or written as x = a, y = b.

Conic Sections

37. In the format $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ (or similar for other sections), the a and b may be left as a^2, b^2 or simplified.

MAML Quick Reference Guide

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Integer	4		4.0 [1] 4.0000 (even on calculator round) [1] $3.\overline{9}$ [1,2] $8/2$ [1,2,4,9] $\begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix}$ [1] $2\csc 30°$ [1] 400% [1] 4.0×10^{0} [1,18] $9-5$ [1,2,4] 2 ^ 2 [1,19] 4.0 E 0 [1,19] 100_{2} [1] – other bases must be requested
Value	$1\frac{1}{7}$	$\frac{8}{7}$ $1.\overline{142857}$ 1.1429 (calc only)	1.142857 [5] $\frac{104}{91}$ [2,9] 1.1429 (non-calc only) [5]
Value	$\frac{2}{3}$	$.\overline{6}$ .6667 (calc only)	$.\overline{66}$ (or any number of 6's >1) [9] $.66\overline{6}$ [9] .6667 (non-calc only) [5] .67 (calc only) [6] .6666 (calc only) [6]
Value - currency and the answer before rounding is 6.2450	$6.25	6.25	6.2450 (calc only) [6] 6.24 [6] $6.24 [6]
Number	$\frac{\sqrt{6}}{2}$	1.2247 (calc only)	$\frac{\sqrt{3}}{\sqrt{2}}$ [11] $\sqrt{\frac{3}{2}}$ [11] $\frac{\sqrt{6}-\sqrt{3}}{2-\sqrt{2}}$ [11]

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Decimal Number rounded to 2 decimal points (Example: the answer before rounding is $1\frac{2}{7}$	1.29		$1\frac{29}{100}$ [1] $1\frac{2}{7}$ [1] $\frac{9}{7}$ [1] 1.2900 or 1.2857 (even on calc round) [1]
Number in base 4	20_4	$20)_4$ 20_{four} 20	8 [1]
Real	π	3.1416 (calc only)	3.14 [5] 3.1415926535898 [5] $\frac{22}{7}$ [5]
Real	$\sqrt{3}$	1.7321 (calc only) $3^{\frac{1}{2}}$	$\frac{3}{\sqrt{3}}$ [11a] $\sqrt[4]{9}$ [11c] $9^{\frac{1}{4}}$ [11d]
Real	$4\left(\sqrt[3]{2}\right)$	$4\cdot 2^{\frac{1}{3}}$ $2^{\frac{7}{3}}$ 5.0397 (calc only)	$\sqrt[3]{128}$ [11b] $\frac{4}{2^{\frac{-1}{3}}}$ [20] $4^{\frac{7}{6}}$ [11d] $2^{\frac{14}{6}}$ [9]
Radical Expression (Example: before simplifying, the answer is $\sqrt{x^2y^3z^4}$)	$\lvert x\rvert yz^2\sqrt{y}$		$\lvert xy\rvert z^2\sqrt{y}$ [11e] $xyz^2\sqrt{y}$ [11e]
Radical Expression (Example: before simplifying, the answer is $\sqrt{(-3x)^2}$)	$3\lvert x\rvert$		$-3x$ [11e] $\lvert -3x\rvert$ [11e]
Complex	$2+4i$	$4i+2$	$\frac{2+4i}{1}$ *or* $\frac{4i+2}{1}$ [13] $(1,2)$ [19] $2(1+2i)$ *or* $2(2i+1)$ [4] $2\sqrt{5}(\cos 63.4349° + i\sin 63.4349°)$ [13] $2\sqrt{5}(\text{cis}\,63.4349°)$ [13]

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Complex	$\frac{15}{2}+\frac{5i}{2}$	$\frac{5i}{2}+\frac{15}{2}$ $\frac{15+5i}{2}$ or $\frac{5i+15}{2}$	$\frac{1}{2}(15+5i)$ *or* $\frac{1}{2}(5i+15)$ [4] $\frac{5}{2}(3+i)$ *or* $\frac{5}{2}(i+3)$ [4] $\frac{5(i+3)}{2}$ *or* $\frac{5(3+i)}{2}$ [4]
Complex	2	$2+0i$ or $0i+2$	
Ordered Pair	$(7,-2)$		7,–2 [14] $(-2,7)$ [14]
Probability	$\frac{\sqrt{3}}{2}$	$50\sqrt{3}\%$ .8660 or 86.6025% (calc only)	$\sqrt{3}:2$ [15] .8660 or 86.6025% (non-calc only) [5]
Probability	$\frac{7}{8}$	87.5% 0.875 0.8750 (calc only)	7:8 or 7 to 8 [15] 7:1 or 7 to 1 [15]
Ratio	$\frac{5}{2}$	5 to 2 5 : 2	$2\frac{1}{2}$ or 2.5 [16] 10 : 4 [16] $\frac{2.5}{1}$ or $2\frac{1}{2}:1$ [16]
Ratio	$\frac{\sqrt{10}}{1}$	$\sqrt{10}:1$ $\sqrt{10}$ to 1	$\sqrt{10}$ [16] $2\sqrt{5}:\sqrt{2}$ or $2\sqrt{5}$ to $\sqrt{2}$ [16]
Angle in the range $0° \le \theta < 360°$	300°	300	–60° [1] $\frac{5\pi}{3}$ [17] $-\frac{\pi}{3}$ [1,17]
Angle in the range $0 \le \theta < \frac{\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{6}$ radians 0.5236 (calc only)	30° [17] $-\frac{11\pi}{6}$ [1]
Algebraic Expression	$4x-7y+2$	{any of the 5 other possible orders of the 3 terms}	$2x+2+y+2x-8y$ [22]
Algebraic Expression	$5\sqrt{3}x+8$	$8+5\sqrt{3}x$	$\sqrt{3}x+8+4\sqrt{3}x$ [22] ($\sqrt{3}x$ is the term)

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Algebraic Expression	$2x^3+10x^2+12x$	$2x(x+3)(x+2)$ or any of the other orders of these three factors	$x(2x^2+10x+12)$ [23]
Algebraic Expression	$2a^2b^2-2a^2c^2+c^4-b^4$	$(b+c)(b-c)$ $(2a^2-b^2-c^2)$	$(b+c)(b-c)((a+b)(a-b)+(a+c)(a-c))$ [23]
Binomial Expansion of $(a+b)^5$	$10a^3b^2$ (3^{rd} term)		$10a^2b^3$ [term number based on descending order of first variable]
Factored Expression	$(c+3)(c-2)$	$(c-2)(c+3)$	$(x+3)(x-2)$ [24]
Solution for y	$y=7x^2-6$	$y=-6+7x^2$ $7x^2-6=y$ $-6+7x^2=y$	$7x^2=y+6$ [1]
Equation	$3a+5=2b$	{any of the 47 other equivalent, simplified forms, including using a 0 term}	$39a+65=26b$ [26]
Equation in specified form	$x^2+y^2=1$	$(x+0)^2+(y+0)^2=1$	
System solution for a, b, and c	$a=2, b=-1, c=3$	{any of the 6 orders of the 3 equations, with or without commas} $(2,-1,3)$	$2,-1,3$ [1,28]
Domain for x	$0<x\le 2$	$x>0$ and $x\le 2$ $\{x: x>0 \text{ and } x\le 2\}$ $\{x: 0<x\le 2\}$ --all 4 of these in either order $(0,2]$	$1<(x+1)\le 3$ [29] $0<3x\le 6$ [29] $x>0$ or $x\le 2$ [1] $x>0\cup x\le 2$ [1] $x>0$, $x\le 2$ [30] **$x>0\cap x\le 2$ [30]** **$\{x: x>0\cap x\le 2\}$ [30]** $[2,0)$

Question asks for a(n)-	And correct answer is -	Also accepted as correct-	All other answers incorrect, including- [see this rule number]
Domain for x	$x > 5$ or $x \le -3$	$\{x : x > 5 \text{ or } x \le -3\}$ $(-\infty,-3]\cup(5,\infty)$ --all 3 in either order	$(-\infty,-3]\cap(5,\infty)$ [1] $x > 5$ and $x \le -3$ [1] **$x > 5 \cup x \le -3$ [30]** **$\{x : x > 5 \cup x \le -3\}$ [30]**
Range of $f(x)$	$f(x) < 2$, except $f(x) \ne 0$	$f(x) < 2$, but $f(x) \ne 0$ $f(x) < 2, f(x) \ne 0$ $f(x) < 2$ and $f(x) \ne 0$ $(-\infty,0)\cup(0,2)$ all reals < 2, except 0 $y < 2, y \ne 0$	$f(x) < 2$ or $f(x) \ne 0$ [1] $(-\infty,0)\cap(0,2)$ [1] $f < 2$, except $f \ne 0$ [1]
Solution such that there are multiple solutions for a single variable	$x = 0$ or $x = 1$ or $x = 5$	$x = 0,1,5$ $x \in \{0,1,5\}$ $x = 0, x = 1, x = 5$ $x = 0$ and 1 and 5	$x = \{0,1,5\}$ [1] – x is not a set a graph on the number line [33]
Matrix	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$	$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$	$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ [35] $\begin{bmatrix} [1 & 2] & [3 & 4] \end{bmatrix}$ [35]
Point on a coordinate plane	$(5,-3)$	$x = 5, y = -3$	$5,-3$ [36] $(-3,5)$ [36]
Conic Sections	$\frac{(x-3)^2}{16} + \frac{(y+2)^2}{25} = 1$	$\frac{(x-3)^2}{4^2} + \frac{(y+2)^2}{5^2} = 1$	